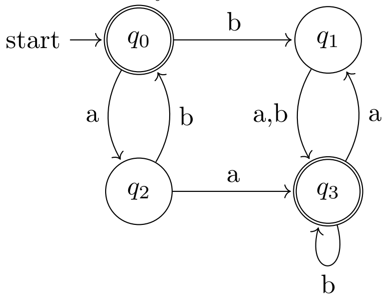
Problems Solved:

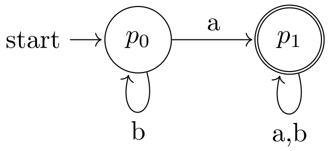
11	12	13	14	15

Name:

Matrikel-Nr.:

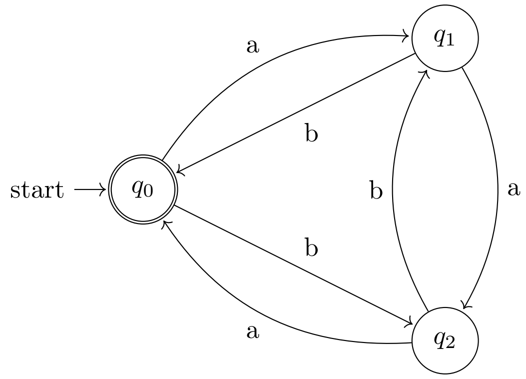
Problem 11. Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be two DFSM over the alphabet Σ. Let $L(M_1)$ and $L(M_2)$ be the languages accepted by M_1 and M_2, respectively.
Construct a DFSM $M = (Q, \Sigma, \delta, q, F)$ whose language $L(M)$ is the intersection of $L(M_1)$ and $L(M_2)$. Write down Q, δ, q, and F explicitly.
Hint: M simulates the parallel execution of M_1 and M_2. For that to work, M "remembers" in its state the state of M_1 as well as the state of M_2. This can be achieved by defining $Q = Q_1 \times Q_2$.
Demonstrate your construction with the following DFSMs.





Problem 12. Let $m > 1$ be a natural number and $r = (a^*b)^m$ be a regular expression.
Definition: If a is a regular expression and n is a natural number, then $a^n = a \cdots a$ is the regular expression that results from the n-fold concatenation of a. For example: $a^3 = aaa$.
Let L_1 and L_2 be the languages defined as follows: $L_1 := L(r)$, $L_2 := \left\{ (a^k b)^m \;\middle|\; k \in \mathbb{N} \right\}$.
Are L_1 and L_2 regular languages? Provide solid arguments to your answers.

Problem 13. Let M_1 be the DFSM with states $\{q_0, q_1, q_2\}$ whose transition graph is given below. Determine a regular expression r such that $L(r) = L(M_1)$. Show the *derivation* of the the final result by the technique based on Arden's Lemma (see lecture notes).



Problem 14. Let r be the following regular expression.

$$a \cdot a \cdot (b \cdot a)^* \cdot b \cdot b^*$$

Construct a nondeterministic finite state machine N such that $L(N) = L(r)$. Show the derivation of the result by following the technique presented in the proof of the theorem *Equivalence of Regular Expressions and Automata* (see lecture notes).

Problem 15. Show that the language $L = \{a^m b^n \mid m, n \in \mathbb{N} \land m \geq 2n\}$ is not regular.