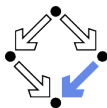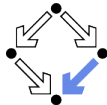# Specifying and Verifying Programs (Part 2)

Wolfgang Schreiner
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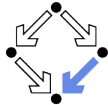
Research Institute for Symbolic Computation (RISC)
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http://www.risc.jku.at

1. Programs as State Relations

2. The RISC ProgramExplorer

Specification by State Predicates

- Hoare calculus and predicate transformers use state predicates.
    - Formulas that talk about a single (pre/post-)state.
    - In such a formula, a reference x means "the value of program variable x in the given state".
- Relationship between pre/post-state is not directly expressible.
    - Requires uninterpreted mathematical constants.

    $$\{x = a\}x := x + 1\{x = a + 1\}$$
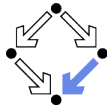
- Unchanged variables have to be explicitly specified.

    $$\{x = a \wedge y = b\}x := x + 1\{x = a + 1 \wedge y = b\}$$

- The semantics of a command c is only implicitly specified.
    - Specifications depend on auxiliary state conditions P, Q.

    $$\{P\}c\{Q\}$$
    $$\text{wp}(c, Q) = P$$

Let us turn our focus from individual states to pairs of states.

Specification by State Relations

- We introduce formulas that denote state relations.
  - Talk about a pair of states (the pre-state and the post-state).
  - old x: "the value of program variable x in the pre-state".
  - var x: "the value of program variable x in the post-state".
- We introduce the logical judgment $c : [F]^{x,\dots}$
  - If the execution of c terminates normally, the resulting post-state is related to the pre-state as described by F.
  - Every variable y not listed in the set of variables $x, \dots$ has the same value in the pre-state and in the post-state.

    $$c : F \wedge \text{var } y = \text{old } y \wedge \dots$$

$x := x + 1 : [\text{var } x = \text{old } x + 1]^x$
$x := x + 1 : \text{var } x = \text{old } x + 1 \wedge \text{var } y = \text{old } y \wedge \text{var } z = \text{old } z \wedge \dots$

We will discuss the termination of commands later.
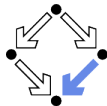
State Relation Rules

$$\frac{c : [F]^{xs} \quad y \notin xs}{c : [F \wedge \text{var } y = \text{old } y]^{xs \cup \{y\}}}$$

$$\textbf{skip} : [\text{true}]^{\emptyset} \qquad \textbf{abort} : [\text{true}]^{\emptyset} \qquad x = e : [\text{var } x = e']^{x}$$

$$\frac{c_1 : [F_1]^{xs} \quad c_2 : [F_2]^{xs}}{c_1; c_2 : [\exists ys : F_1[ys/\text{var } xs] \wedge F_2[ys/\text{old } xs]]^{xs}}$$

$$\frac{c : [F]^{xs}}{\textbf{if } e \textbf{ then } c : [\text{if } e' \text{ then } F \text{ else var } xs = \text{old } xs]^{xs}}$$
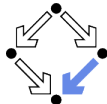
$$\frac{c_1 : [F_1]^{xs} \quad c_2 : [F_2]^{xs}}{\textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 : [\text{if } e' \text{ then } F_1 \text{ else } F_2]^{xs}}$$

$$c : [F]^{xs}$$
$$\vdash \forall xs, ys, zs : I[xs/\text{old } xs, ys/\text{var } xs] \wedge e[ys/xs] \wedge F[ys/\text{old } xs, zs/\text{var } xs] \Rightarrow$$
$$I[xs/\text{old } xs, zs/\text{var } xs]$$
$$\overline{\textbf{while } e \textbf{ do } \{I, t\} \ c : [\neg e'' \wedge (I[\text{old } xs/\text{var } xs] \Rightarrow I)]^{xs}}$$

if e then F_1 else $F_2 :\Leftrightarrow (e \Rightarrow F_1) \wedge (\neg e \Rightarrow F_2)$
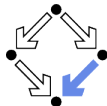$e' := e[\text{old } xs/xs], e'' := e[\text{var } xs/xs]$ (for all program variables xs)

Example
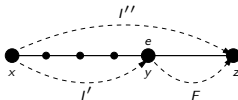
$c_1 = y := y + 1;$
$c_2 = x := x + y$

$c_1 : [\text{var } y = \text{old } y + 1]^y$
$c_2 : [\text{var } x = \text{old } x + \text{old } y]^x$

$c_1 : [\text{var } y = \text{old } y + 1 \land \text{var } x = \text{old } x]^{x,y}$
$c_2 : [\text{var } x = \text{old } x + \text{old } y \land \text{var } y = \text{old } y]^{x,y}$

$c_1; c_2 : [\exists x_0, y_0 :$
$\qquad y_0 = \text{old } y + 1 \land x_0 = \text{old } x \land$
$\qquad \text{var } x = x_0 + y_0 \land \text{var } y = y_0]^{x,y}$

$c_1; c_2 : [\text{var } x = \text{old } x + \text{old } y + 1 \land \text{var } y = \text{old } y + 1]^{x,y}$

Mechanical translation and logical simplification.

Loops

$$c : [F]^{xs}$$
$$\vdash \forall xs, ys, zs : I[xs/\text{old } xs, ys/\text{var } xs] \wedge e[ys/xs] \wedge F[ys/\text{old } xs, zs/\text{var } xs] \Rightarrow$$
$$I[xs/\text{old } xs, zs/\text{var } xs]$$
$$\overline{\textbf{while } e \textbf{ do } \{I, t\}\ c : [\neg e'' \wedge (I[\text{old } xs/\text{var } xs] \Rightarrow I)]^{xs}}$$

$w = \textbf{while } i < n \textbf{ do } \{I, t\}\ (s := s + i; i := i + 1)$
$I \Leftrightarrow 0 \leq \text{var } i \leq \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{var } i - 1} j$

$(s := s + i; i := i + 1) : [\text{var } s = \text{old } s + \text{old } i \wedge \text{var } i = \text{old } i + 1]^{s,i}$
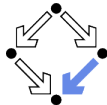$\vdash \forall s_x, s_y, s_z, i_x, i_y, i_z :$
$\quad (0 \leq i_y \leq \text{old } n \wedge s_y = \sum_{j=0}^{i_y - 1} j) \wedge i_y < \text{old } n \wedge (s_z = s_y + i_y \wedge i_z = i_y + 1) \Rightarrow$
$\quad \underline{0 \leq i_z \leq \text{old } n \wedge s_z = \sum_{j=0}^{i_z - 1} j}$
$w : [\neg(\text{var } i < \text{var } n) \wedge (0 \leq \text{old } i \leq \text{old } n \wedge \text{old } s = \sum_{j=0}^{\text{old } i - 1} j \Rightarrow I)]^{s,i}$

The loop relation is derived from the invariant (not the loop body); we have to prove the preservation of the loop invariant.
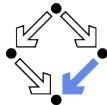
Example

$$c =$$
    if $n < 0$
        $s := -1$
    else
        $s := 0$
        $i := 0$
        while $i < n$ **do** $\{l,t\}$
           $s := s + i$
           $i := i + 1$

$l \Leftrightarrow 0 \le \text{var } i \le \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{var } i-1} j$
$t = \text{old } n - \text{old } i$

$c : [\text{if old } n < 0$
    $\text{then var } i = \text{old } i \wedge \text{var } s = -1$
    $\text{else var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$

Let us calculate this "semantic essence" of the program.

Example

$c = \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b$
$b = (s := 0; i := 0; w)$
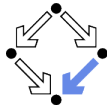$w = \textbf{while } i < n \textbf{ do } \{I, t\} \; (s := s + i; i := i + 1)$

$s := 0 : [\text{var } s = 0]^s$
$s := 0 : [\text{var } s = 0 \land \text{var } i = \text{old } i]^{s,i}$

$i := 0 : [\text{var } i = 0]^i$
$i := 0 : [\text{var } i = 0 \land \text{var } s = \text{old } s]^{s,i}$

$s := 0; i := 0 : [\exists s_0, i_0 : s_0 = 0 \land i_0 = \text{old } i \land \text{ var } i = 0 \land \text{var } s = s_0]^{s,i}$
$s := 0; i := 0 : [\text{var } s = 0 \land \text{var } i = 0]^{s,i}$

$w : [\neg(\text{var } i < \text{var } n) \land (0 \le \text{old } i \le \text{old } n \land \text{old } s = \sum_{j=0}^{\text{old } i - 1} j \Rightarrow I)]^{s,i}$
$w : [\text{var } i \ge \text{old } n \land (0 \le \text{old } i \le \text{old } n \land \text{old } s = \sum_{j=0}^{\text{old } i - 1} j \Rightarrow I)]^{s,i}$

Example

$c = \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b$
$b = (s := 0; i := 0; w)$
$w = \textbf{while } i < n \textbf{ do } \{I, t\} \ (s := s + i; i := i + 1)$

$s := 0; i := 0 : [\text{var } s = 0 \wedge \text{var } i = 0]^{s,i}$
$w : [\text{var } i \geq \text{old } n \wedge (0 \leq \text{old } i \leq \text{old } n \wedge \text{old } s = \sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}$

$b : [\exists s_0, i_0 : s_0 = 0 \wedge i_0 = 0 \wedge$
$\quad \text{var } i \geq \text{old } n \wedge (0 \leq i_0 \leq \text{old } n \wedge s_0 = \sum_{j=0}^{i_0-1} j \Rightarrow I)]^{s,i}$
$b : [\exists s_0, i_0 : s_0 = 0 \wedge i_0 = 0 \wedge$
$\quad \text{var } i \geq \text{old } n \wedge (0 \leq \text{old } n \Rightarrow I)]^{s,i}$
$b : [\text{var } i \geq \text{old } n \wedge$
$\quad (0 \leq \text{old } n \Rightarrow 0 \leq \text{var } i \leq \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{var } i-1} j)]^{s,i}$
$b : [\text{var } i \geq \text{old } n \wedge$
$\quad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$

Example
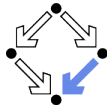
$c = \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b$
$b = (s := 0; i := 0; w)$
$w = \textbf{while } i < n \textbf{ do } \{l, t\} \ (s := s + i; i = i + 1)$
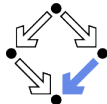
$s := -1 : [\text{var } s = -1]^s$
$s := -1 : [\text{var } i = \text{old } i \wedge \text{var } s = -1]^{s,i}$

$b : [\text{var } i \geq \text{old } n \ \wedge$
$\quad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$

$c : [\textbf{if old } n < 0$
$\quad \text{then var } i = \text{old } i \wedge \text{var } s = -1$
$\quad \text{else var } i \geq \text{old } n \ \wedge$
$\qquad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$
$c : [\textbf{if old } n < 0$
$\quad \text{then var } i = \text{old } i \wedge \text{var } s = -1$
$\quad \text{else var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$

Partial Correctness

- Specification (xs, P, Q)
    - Set of program variables xs (which may be modified).
    - Precondition P (a formula with "old xs" but no "var xs").
    - Postcondition Q (a formula with both "old xs" and "var xs").
- Partial correctness of implementation c
    1. Derive $c : [F]^{xs}$.
    2. Prove $F \Rightarrow (P \Rightarrow Q)$
        - Or: $P \Rightarrow (F \Rightarrow Q)$
        - Or: $(P \wedge F) \Rightarrow Q$

Verification of partial correctness leads to the proof of an implication.

Relationship to Other Calculi
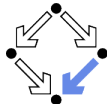
Let all state conditions refer via "*old* xs" to program variables *xs*.

- **Hoare Calculus**
    - For proving $\{P\}c\{Q\}$,
    - it suffices to derive $c : [F]^{xs}$
    - and prove $P \wedge F \Rightarrow Q[\text{var } xs/\text{old } xs]$.
- **Predicate Transformers**
    - Assume we can derive $c : [F]^{xs}$.
    - If c does not contain loops, then
      $$\text{wp}(c, Q) = \forall xs : F[xs/\text{var } xs] \Rightarrow Q[xs/\text{old } xs]$$
      $$\text{sp}(c, P) = \exists xs : P[xs/\text{old } xs] \wedge F[xs/\text{old } xs, \text{old } xs/\text{var } xs]$$
    - If c contains loops, the result is still a valid pre/post-condition but not necessarily the weakest/strongest one.

A generalization of the previously presented calculi.

Termination
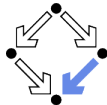
- We introduce a judgment $c \downarrow T$.

    State condition T (a formula with "old xs" but no "var xs").

    - Starting with a pre-state that satisfies condition T the execution of command c terminates.

- Total correctness of implementation c.

    Specification (xs, P, Q).

    - Derive $c \downarrow T$.
    - Prove $P \Rightarrow T$.

Also verification of termination leads to the proof of an implication.
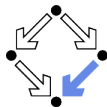
Termination Condition Rules

$$\textbf{skip} \downarrow \text{true} \qquad\qquad \textbf{abort} \downarrow \text{true} \qquad\qquad x := e \downarrow \text{true}$$

$$\frac{c_1 \downarrow T_1 \qquad c_2 \downarrow T_2}{c_1; c_2 \downarrow T_1 \wedge \text{wp}(c_1, T_2)}$$

$$\frac{c \downarrow T}{\textbf{if } e \textbf{ then } c \downarrow e' \Rightarrow T}$$

$$\frac{c_1 \downarrow T_1 \qquad c_2 \downarrow T_2}{\textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 \downarrow \text{if } e' \text{ then } T_1 \text{ else } T_2}$$

$$c : [F]^{xs} \qquad c \downarrow T$$

$$\frac{\begin{array}{l} \vdash \forall xs, ys, zs : \\ \quad I[xs/\text{old } xs, ys/\text{var } xs] \wedge e[ys/xs] \wedge F[ys/\text{old } xs, zs/\text{var } xs] \wedge t[ys/\text{old } xs] \geq 0 \Rightarrow \\ \quad\quad T[ys/\text{old } xs] \wedge 0 \leq t[zs/\text{old } xs] < t[ys/\text{old } xs] \end{array}}{\textbf{while } e \textbf{ do } \{I, t\} \; c \downarrow t \geq 0}$$

In every iteration of a loop, the loop body must terminate and the termination term must decrease (but not become negative).

Example

$$c =$$
    if $n < 0$
        $s := -1$
    else
        $s := 0$
        $i := 0$
        while $i < n$ **do** $\{l,t\}$
            $s := s + i$
            $i := i + 1$
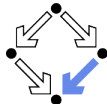
$l \Leftrightarrow 0 \leq$ var $i \leq$ old $n \wedge$ var $s = \sum_{j=0}^{\text{var } i-1} j]$
$t =$ old $n -$ old i

$c \downarrow$ if old $n < 0$ then true else $\ldots$
$c \downarrow$ if old $n < 0$ then true else old $n \geq 0$
$c \downarrow$ true

We still have to prove the constraint on the loop iteration.

Example
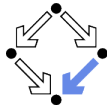
$s := s + i; i := i + 1 \downarrow \text{true}$

$\forall s_x, s_y, s_z, i_x, i_y, i_z :$
$\quad (0 \le i_y \le \text{old } n \wedge s_y = \sum_{j=0}^{i_y - 1} j) \wedge$
$\quad i_y < \text{old } n \wedge$
$\quad (s_z = s_y + i_y \wedge i_z = i_y + 1) \wedge$
$\quad \text{old } n - i_y \ge 0 \Rightarrow$
$\quad\quad \text{true} \wedge$
$\quad\quad 0 \le \text{old } n - i_z < \text{old } n - i_y$

Also this constraint is simple to prove.

Abortion
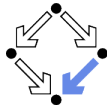
Also abortion can be ruled out by proving side conditions in the usual way.

Wolfgang Schreiner. *Computer-Assisted Program Reasoning Based on a Relational Semantics of Programs.* Research Institute for Symbolic Computation (RISC), Johannes Kepler University, Linz, Austria, 2011.

See the report for the full calculus.
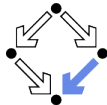
1. **Programs as State Relations**

2. **The RISC ProgramExplorer**

The RISC ProgramExplorer
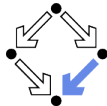
- An integrated environment for program reasoning.
  - Research Institute for Symbolic Computation (RISC), 2008–.
    - http://www.risc.jku.at/research/formal/software/ProgramExplorer
      - Integrates the RISC ProofNavigator for computer-assisted proving.
      - Written in Java, runs under Linux (only), freely available (GPL).
- Programs written in "MiniJava".
  - Subset of Java with full support of control flow interruptions.
  - Value (not pointer) semantics for arrays and objects.
- Theories and specifications written in a formula language.
  - Derived from the language of the RISC ProofNavigator.
- Semantic analysis and verification.
  - Program methods are translated into their "semantic essence".
    - Open for human inspection.
  - From the semantics, the verification tasks are generated.
    - Solved by automatic decision procedure or interactive proof.

Tight integration of executable programs, declarative specifications, mathematical semantics, and verification tasks.

Using the Software
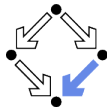
See "The RISC ProgramExplorer: Tutorial and Manual".

- Develop a theory.
    - File "*Theory*.theory" with a theory *Theory* of mathematical types, constants, functions, predicates, axioms, and theorems.
    - Can be also added to a program file.
- Develop a program.
    - File "*Class*.java" with a class *Class* that contains class (static) and object (non-static) variables, methods and constructors.
    - Class may be annotated by a theory (and an object invariant).
    - Methods may be annotated by method specifications.
    - Loops may be annotated by invariants and termination terms.
- Analyze method semantics.
    - Transition relations, termination conditions, ... of the method body and its individual commands.
- Perform verification tasks.
    - Frame, postcondition, termination, preconditions, loop-related tasks, type-checking conditions.

Starting the Software
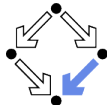
- Starting the software:
  ```
  module load ProgramExplorer (only at RISC)
  ProgramExplorer &
  ```
- Command line options:
  ```
  Usage: ProgramExplorer [OPTION]...
  OPTION: one of the following options:
      -h, --help: print this message.
      -cp, --classpath [PATH]:
        directories representing top package.

  Environment Variables:
      PE_CLASSPATH:
          the directories (separated by ":") representing the
          top package (default the current working directory)
      ...
  ```
- Task repository created/read in current working directory:
  ```
  Subdirectory .PETASKS.timestamp (ProgramExplorer tasks)
  Subdirectory .ProofNavigator (ProofNavigator legacy)
  ```

The Graphical User Interface
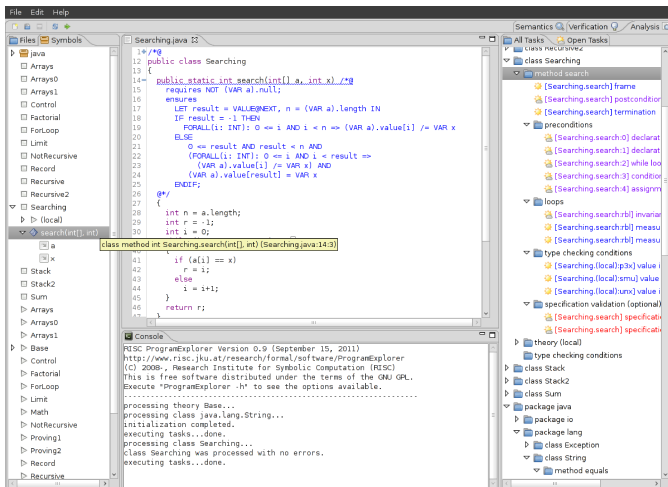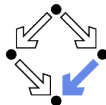
A Program
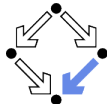
```
/*@..
class Sum
{
  static int sum(int n) /*@..
  {
    int s;
    if (n < 0)
      s = -1;
    else
    {
      s = 0;
      int i = 1;
      while (i <= n) /*@..
      {
        s = s+i;
        i = i+1;
      }
    }
    return s;
  }
}
```

Markers /*@.. indicate
hidden mathematical annotations.

A Theory
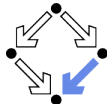
```
/*@
  theory {
    sum: (INT, INT) -> INT;
    sumaxiom: AXIOM
      FORALL(m: INT, n: INT):
        IF n<m THEN
          sum(m, n) = 0
        ELSE
          sum(m, n) = n+sum(m, n-1)
        ENDIF;
  }
@*/
class Sum
...
```

The introduction of a function $sum(m, n) = \sum_{j=m}^{n} j$.
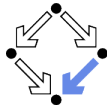
A Method Specification

```
static int sum(int n) /*@
  requires VAR n < Base.MAX_INT;
  ensures
    LET result=VALUE@NEXT IN
    IF VAR n < 0
      THEN result = -1
      ELSE result = sum(1, VAR n)
    ENDIF;
@*/
...
```

For non-negative *n*, a call of program method sum(*n*) returns *sum*(1, *n*) (and does not modify any global variable).

A Loop Annotation
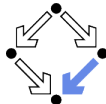
```
while (i <= n) /*@
  invariant VAR n < Base.MAX_INT
        AND 1 <= VAR i AND VAR i <= VAR n+1
        AND VAR s=sum(1, VAR i-1);
  decreases VAR n - VAR i + 1;
@*/
{
  s = s+i;
  i = i+1;
}
}
```

The loop invariant and termination term (measure).

The Specification Language
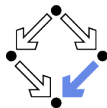
Derived from the language of the RISC ProofNavigator.

- State conditions/relations, state terms.
    - State condition: method precondition (`requires`).
    - State relation: method postcondition (`ensures`),
      loop invariant (`invariant`).
    - State term: termination term (`decreases`).
- References to program variables.

    `OLD` x: the value of program variable x in the pre-state.

    `VAR` x: the value of program variable x in the post-state.

    - In state conditions/terms, both refer to the value in the current state.
    - If program variable is of the program type T, then then `OLD`/`VAR` x is
      of the mathematical type T'.

        `int` $\rightarrow$ `Base.int` = `[Base.MIN_INT,Base.MAX_INT]`.
- Function results

    `VALUE@NEXT`: the return value of a program function.

    - The value of the function call's post-state `NEXT`.

The Semantics View

The Method Body

Body Knowledge

[Show Original Formulas]

Pre-State Knowledge

old $n < \text{Base.MAX}_{\text{INT}}$

Effects

executes: false, **continues:** false, **breaks:** false, **returns:** true
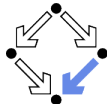**variables:** -; **exceptions:**-

Transition Relation

if old $n < 0$ then
  returns@next $\wedge$ value@next $= -1$
else
   returns@next
   $\wedge$
   $(\exists \text{in} \in \text{Base.int: in} = \text{old } n + 1 \wedge 1 \leq \text{in} \wedge \text{value@next} = \text{sum}(1, \text{in} - 1))$
   $\wedge$
   old $n < \text{Base.MAX}_{\text{INT}}$
endif

Select method symbol "sum" and menu entry "Show Semantics".

A Body Command

Statement Knowledge

[Show Original Formulas]

Pre-State Knowledge

$\text{old } n < \text{Base.MAX}_{\text{INT}} \land \text{old } n \geq 0 \land \text{old } s = 0 \land \text{old } i = 1$

Precondition

$\text{old } n < \text{Base.MAX}_{\text{INT}} \land 1 \leq \text{old } i \land \text{old } i \leq \text{old } n + 1 \land \text{old } s = \text{sum}(1, \text{old } i - 1)$

Effects

executes: true, **continues:** false, **breaks:** false, **returns:** false
variables: s, i; **exceptions:**-

Transition Relation

$\text{var } i = \text{old } n + 1 \land \text{old } n < \text{Base.MAX}_{\text{INT}} \land 1 \leq \text{var } i \land \text{var } s = \text{sum}(1, \text{var } i - 1)$

Termination Condition

$\text{executes@now} \Rightarrow \text{old } n - \text{old } i \geq -1$

Move the mouse pointer
over the box to the
left of the loop.

The Semantics Elements
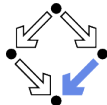
- **Pre-State Knowledge**
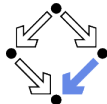   What is known about the pre-state of the command.
- **Precondition**
    What has to be true for the pre-state of the command such that
    the command may be executed.
- **Effects**
    Which kind of effects may the command have.
    - **variables**: which variables may be changed.
    - **exceptions**: which exceptions may be thrown.
    - **executes**, **continues**, **breaks**, **returns**: may the execution terminate
      normally, may it be terminated by a `continue`, `break`, `return`.
- **Transition Relation**
    The prestate/poststate relationship of the command.
- **Termination**
    What has to be true for the pre-state of the command such that
    the command terminates.

Formulas are shown after simplification (see "Show Original Formulas").

Constraining a State

Select the loop body, enter in the box
the condition VAR s=2 AND VAR i=1,
press "Submit", and move the mouse to i=i+1.

State Conditions

[Show Original Formulas]

Pre-State Condition

$\mathrm{var}\ i = 1 \wedge \mathrm{var}\ s = \mathrm{var}\ i + 2$

Post-State Condition

$\mathrm{var}\ s = 3 \wedge \mathrm{var}\ i = 2$

The Verification Tasks

class Sum
  method sum
    [Sum.sum] effects
    [Sum.sum] postcondition
    [Sum.sum] termination
    preconditions
      [Sum.sum:0] assignment precondition
      [Sum.sum:1] while loop precondition
      [Sum.sum:2] assignment precondition
      [Sum.sum:3] assignment precondition
    loops
      [Sum.sum:qvb] invariant is preserved
      [Sum.sum:qvb] measure is well-formed
      [Sum.sum:qvb] measure is decreased
    type checking conditions
    specification validation (optional)
      [Sum.sum] specification is satisfiable
      [Sum.sum] specification is not trivial

The Verification Tasks

- **Effects:** does the method only change those global variables indicated in the method's `assignable` clause?
- **Postcondition:** do the method's precondition and the body's state relation imply the method's postcondition?
- **Termination:** does the method's precondition imply the body's termination condition?
- **Precondition:** does a statement's prestate knowledge imply the statement's precondition?
- **Loops:** is the loop invariant preserved, the measure well-formed (does not become negative) and decreased?
- **Type checking conditions:** are all formulas well-typed?
- **Specification validation:** does for every input that satisfies a precondition exist a result that does (not) satisfy the postcondition?

Partially solved by automatic decision procedure, partially by an interactive computer-supported proof.

The Task States
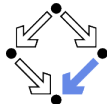
The task status is indicated by color (icon).

- Blue (sun): the task was solved in the current execution of the RISC ProgramExplorer (automatically or by an interactive proof).
- Violet (partially clouded): the task was solved in a previous execution by an interactive proof.
    - Nothing has changed, so we need not perform the proof again.
    - However, we may replay the proof to investigate it.
- Red (partially clouded): there exists a proof but it is either not complete or cannot be trusted any more (something has changed).
- Red (fully clouded): there does not yet exist a proof.

Select "Execute Task" to start/replay a proof, "Show Proof" to display a proof, "Reset Task" to delete a proof.

A Postcondition Proof

Linear Search

```
/*@..
public class Searching
{
  public static int search(int[] a, int x) /*@..
  {
    int n = a.length;
    int r = -1;
    int i = 0;
    while (i < n && r == -1) /*@..
    {
      if (a[i] == x)
        r = i;
      else
        i = i+1;
    }
    return r;
  }
}
```

The Representation of Arrays
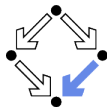
The program type int[] is mapped to the mathematical type
Base.IntArray.

```
theory Base
{
  ...
  IntArray: TYPE =
    [#value: ARRAY int OF int, length: nat, null: BOOLEAN#];
  ...
}
```

- (VAR *a*).length: the number of elements in array *a*.
- (VAR *a*).value[*i*]: the element with index *i* in array *a*.
- (VAR *a*).null: *a* is the null pointer.

Program type *Class* is mapped to mathematical type *Class.Class*;
Class[] is mapped to *Class.Array*.
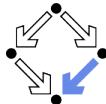
Theory

```
/*@
theory uses Base {
  int: TYPE = Base.int;
  intArray: TYPE = Base.IntArray;
  smallestPosition: FORMULA
    FORALL(a: intArray, n: NAT, x: int):
      (EXISTS(i:int): 0 <= i AND i < n AND a.value[i] = x) =>
      (EXISTS(i:int): 0 <= i AND i < n AND a.value[i] = x AND
         (FORALL(j:int): 0 <= j AND j < n AND a.value[j] = x =>
           j >= i));
}
@*/
public class Searching
...
```

Method Specification
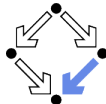
```
public static int search(int[] a, int x) /*@
  requires (VAR a).null = FALSE;
  ensures
    LET result = VALUE@NEXT, n = (VAR a).length IN
    IF result = -1 THEN
      FORALL(i: INT): 0 <= i AND i < n =>
        (VAR a).value[i] /= VAR x
    ELSE
      0 <= result AND result < n AND
      (FORALL(i: INT): 0 <= i AND i < result =>
        (VAR a).value[i] /= VAR x) AND
      (VAR a).value[result] = VAR x
    ENDIF;
@*/
...
```

Loop Annotation
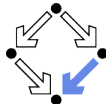
```
while (i < n && r == -1) /*@
  invariant (VAR a).null = FALSE AND VAR n = (VAR a).length
        AND 0 <= VAR i AND VAR i <= VAR n
        AND (FORALL(i: INT): 0 <= i AND i < VAR i =>
              (VAR a).value[i] /= VAR x)
        AND (VAR r = -1 OR (VAR r = VAR i AND VAR i < VAR n AND
              (VAR a).value[VAR r] = VAR x));
  decreases IF VAR r = -1 THEN VAR n - VAR i ELSE 0 ENDIF;
@*/
{
  if (a[i] == x)
    r = i;
  else
    i = i+1;
}
```
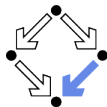
Method Semantics
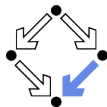
Transition Relation

$(\exists \text{in} \in \text{Base.int}, \ n \in \text{Base.int}:$

$\quad n = \text{old } a.\text{length} \land (\text{in} \geq n \lor \text{value@next} \neq -1) \land 0 \leq \text{in} \land \text{in} \leq n$

$\quad \land$

$\quad (\forall i \in \mathbb{Z}: 0 \leq i \land i < \text{in} \Rightarrow \text{old } a.\text{value}[i] \neq \text{old } x)$

$\quad \land$

$\quad (\ \text{value@next} = -1$

$\quad \ \lor$

$\quad \quad \text{value@next} = \text{in} \land \text{in} < n \land \text{old } a.\text{value}[\text{value@next}] = \text{old } x)) \land \neg \text{ old } a.\text{null}$

$\land$

$\quad \text{returns@next}$

Termination Condition

$\text{executes@now} \Rightarrow \text{old } a.\text{length} \geq 0$

Verification Tasks

▽ 📁 method search
    ☀ [Searching.search] effects
    ☀ [Searching.search] postcondition
    ☀ [Searching.search] termination
  ▽ 📁 preconditions
    ☀ [Searching.search:0] declaration precondition
    ☀ [Searching.search:1] declaration precondition
    ☀ [Searching.search:2] while loop precondition
    ☀ [Searching.search:3] conditional precondition
    ☀ [Searching.search:4] assignment precondition
  ▽ 📁 loops
    ☀ [Searching.search:rbl] invariant is preserved
    ☀ [Searching.search:rbl] measure is well-formed
    ☀ [Searching.search:rbl] measure is decreased
  ▽ 📁 type checking conditions
    ☀ [Searching.(local):p3x] value is in interval
    ☀ [Searching.(local):smu] value is in interval
    ☀ [Searching.(local):unx] value is in interval
  ▷ 📁 specification validation (optional)

Invariant Proof

Working Strategy

- Develop theory.
    - Introduce interesting theorems that may be used in verifications.
- Develop specifications.
    - Validate specifications, e.g. by showing satisfiability and non-triviality.
- Develop program with annotations.
    - Validate programs/annotations by investigating program semantics.
- Prove postcondition and termination.
    - Partial and total correctness.
    - By proofs necessity of additional theorems may be detected.
- Prove precondition tasks and loop tasks.
    - By proofs necessity of additional theorems may be detected.
- Prove mathematical theorems.
    - Validation of auxiliary knowledge used in verifications.

The integrated development of theories, specifications, programs, annotations is crucial for the design of provably correct programs.