

**Problems Solved:**

31	32	33	34	35
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**Problem 31.** Let a language  $L = L(T) \subseteq \{0, 1\}^*$  be given by the code of a Turing machine  $\langle T \rangle$ . It is known that  $\varepsilon \in L$ .

Let  $S_0$  be the set of Turing machines of the form  $(Q, \{0, 1, X, \sqcup\}, \sqcup, \{0, 1\}, q_0, \emptyset)$ . Let  $S_1$  be the set of Turing machines of the form  $(Q, \{0, 1, X, \sqcup\}, \sqcup, \{0, 1\}, q_0, Q)$ . Is it decidable whether  $L = L(M)$  and  $M \in S_0$ ? That is: Is there a Turing machine  $D_0$  such that it takes a word  $w$  as input and returns “yes” if  $w = \langle M \rangle$  for a TM  $M \in S_0$  with the property  $L(M) = L$ , and returns “no” otherwise? What is the answer, if you replace  $S_0$  by  $S_1$ ? Justify your answers.

**Problem 32.** Let  $\Sigma$  be an alphabet and  $A$  be a language over  $\Sigma$  ( $A \subseteq \Sigma^*$ ). Let also  $\bar{A}$  be semi-decidable, but not decidable. Prove that the complement of  $A$ , i. e.,  $\bar{A} = \Sigma^* \setminus A$ , is not decidable.

**Problem 33.** Let  $L$  be a finite language over an alphabet  $\{0, 1\}$ . Is the following problem (with input  $\langle M \rangle$ )

For a Turing machine  $M$  it holds  $L(M) \supseteq L$ .

in general semi-decidable? Is it also in general decidable? Justify your answers.

**Problem 34.** Which of the following problems are decidable? In each problem below, the input of the problem is the code  $\langle M \rangle$  of a Turing machine  $M$  with input alphabet  $\{0, 1\}$ .

- (a) Does  $M$  have at least 4 states?
- (b) Is  $L(M) \subseteq \{0, 1\}^*$ ?
- (c) Is  $L(M)$  recursive?
- (d) Is  $L(M)$  finite?
- (e) Is  $10101 \in L(M)$ ?
- (f) Is  $L(M)$  not recursively enumerable?
- (g) Does there exist a word  $w \in L(M)$  such that  $M$  does not halt on  $w$ ?

Justify your answer.

**Problem 35.** Show that the Acceptance Problem is reducible to the restricted Halting problem. First explain clearly which Turing machine you have to construct to prove this statement and then give a reasonably detailed description of this construction.