Problems Solved:

16	17	18	19	20

Name:

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Problem 16. Answer the following questions and provide reasons for your answers. You need to be concerned about the implications for the general computability of functions.

1. Let R be a RAM that reads exactly one number from its input tape and always terminates with 0 or 1 written on its output tape. Is there a function $f : \mathbb{N} \to \mathbb{N}$ such that $f(x) = y$ if and only if the input was x and after termination y is on the output tape of R?
2. Let $f : \mathbb{N} \to \mathbb{N}$ be a function. Is there always a RAM R such that R terminates on every input and that R with input $n \in \mathbb{N}$ has written $f(n)$ to its output tape?

Hint: The task basically asks you whether there is a bijection between the set of functions from $\mathbb{N}$ to $\mathbb{N}$ and the set of all RAMs. For one part you can use a cardinality argument for those sets.

Problem 17. Write a RAM program that from a given natural number n prints its binary representation. In order to simplify the problem the output shall be in low positions first format, i.e., the number 8_{10} is 0001_2 but not 1000_2.
Hint: please note that the computation of the quotient respectively remainder of a division by 2 can be implemented by the repeated subtraction of 2.

Problem 18. In the following use *only* the definition of a *loop program* as given in Def. 23 of the lecture notes, Section 3.2.2. Note that it is not allowed to use abbreviations like

```
xi := xj − xk;
xi := xj + xk;
```

Furthermore, the variables in a loop program are only $x_0, x_1, \ldots$

1. Show that the function

$$s(x_1, x_2) = \begin{cases} 1 & \text{if } x_1 < x_2, \\ 0 & \text{otherwise} \end{cases}$$

   is loop computable. I.e. give an explicit loop program for s.

2. Write a loop program that computes the function $d : \mathbb{N}^2 \to \mathbb{N}$ where $d(x_1, x_2)$ is $k \in \mathbb{N}$ such that $k \cdot (x_2 + 1) = x_1 + 1$ if such a k exists. The result is $d(x_1, x_2) = 0$, if a k with the above property does not exist.

   For simplicity in the program for d, you are allowed to use a construction like the following (with the obvious semantics) where P is an arbitrary loop program.

   IF $x_i < x_j$ **THEN** P **END**;

   Note: Only $<$ is allowed in the condition and there is no “ELSE” branch.

Problem 19. Provide a loop program that computes the function $f(n) = n! = \prod_{k=1}^{n} k$, and thus show that f is loop computable. In your program you are only allowed to uses statements from Definition 23 of the lecture notes.

Problem 20. Suppose P is a while-program that does not contain any `WHILE` statements, but might modify the values of the variables x_1 and x_2.
Transform the following program into Kleene's normal form.
Hint: first translate the program into a goto program, replace the `GOTO`s by assignments to a control variable, and add the `WHILE` wrapper.

```
x0 := 0
WHILE x1 DO
    x1 := x1 - 1;
    x2 := x1;
    WHILE x2 DO
        P;
    END;
END;
x0 := x0 + 1
```