

Figure 7.5

Abstract syntax:

$E \in$ Expression
 $A \in$ Atomic-symbol
 $I \in$ Identifier

$E ::= LET\ I = E_1\ IN\ E_2\ | LAMBDA\ (I)\ E_1\ E_2\ |$

$E_1\ CONS\ E_2\ | HEAD\ E\ | TAIL\ E\ | NIL\ | I\ | A\ | (E)$

Semantic algebras:

I. Atomic answer values

Domain $a \in Atom$

Operations

(omitted)

II. Identifiers

Domain $i \in Id = Identifier$

Operations

(usual)

III. Denotable values, functions, and lists

Domains $d \in Denotable-value = (Function + List + Atom + Error)_\perp$

$f \in Function = Denotable-value \rightarrow Denotable-value$

$t \in List = Denotable-value^*$

$Error = Unit$

IV. Expressible values

Domain $x \in Expressible-value = Denotable-value$

V. Environments

Domain $e \in Environment = Id \rightarrow Denotable-value$

Operations

$accessenv : Id \rightarrow Environment \rightarrow Denotable-value$

$accessenv = \lambda i.\lambda e. e(i)$

$updateenv : Id \rightarrow Denotable-value \rightarrow Environment \rightarrow Environment$

$updateenv = \lambda i.\lambda d.\lambda e. [i \mapsto d]e$

Valuation functions:

$E : Expression \rightarrow Environment \rightarrow Expressible-value$

$E[LET\ I = E_1\ IN\ E_2] = \lambda e. E[E_2](updateenv\ [I]\ (E[E_1]e))$

$E[LAMBDA\ (I)\ E] = \lambda e. inFunction(\lambda d. E[E](updateenv\ [I]\ d\ e))$

Figure 7.5 (continued)

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E[E1 E2] = λe. let x = (E[E1]e) in cases x of
  isFunction(f) → f(E[E2]e)
  isList(t) → inError()
  isAtom(a) → inError() [] isError() → inError() end

E[E1 CONS E2] = λe. let x = (E[E2]e) in cases x of
  isFunction(f) → inError()
  isList(t) → inList(E[E1]e cons t)
  isAtom(a) → inError() [] isError() → inError() end

E[HEAD E] = λe. let x = (E[E]e) in cases x of
  isFunction(f) → inError()
  isList(t) → (null t → inError() [] (hd t))
  isAtom(a) → inError() [] isError() → inError() end

E[TAIL E] = λe. let x = (E[E]e) in cases x of
  isFunction(f) → inError()
  isList(t) → (null t → inError() [] inList(tl t))
  isAtom(a) → inError() [] isError() → inError() end

E[NIL] = λe. inList(nil)
E[I] = accessenv [I]
E[A] = λe. inAtom(A[A])
E[E] = E[E]
A: Atom-symbol → Atom (omitted)

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Let $E_0 = \text{LET } F = a_0 \text{ IN } E_1$
 $E_1 = \text{LET } F = \text{LAMBDA } (Z) \text{ F CONS } Z \text{ IN } E_2$
 $E_2 = \text{LET } Z = a_1 \text{ IN F(Z CONS NIL)}$

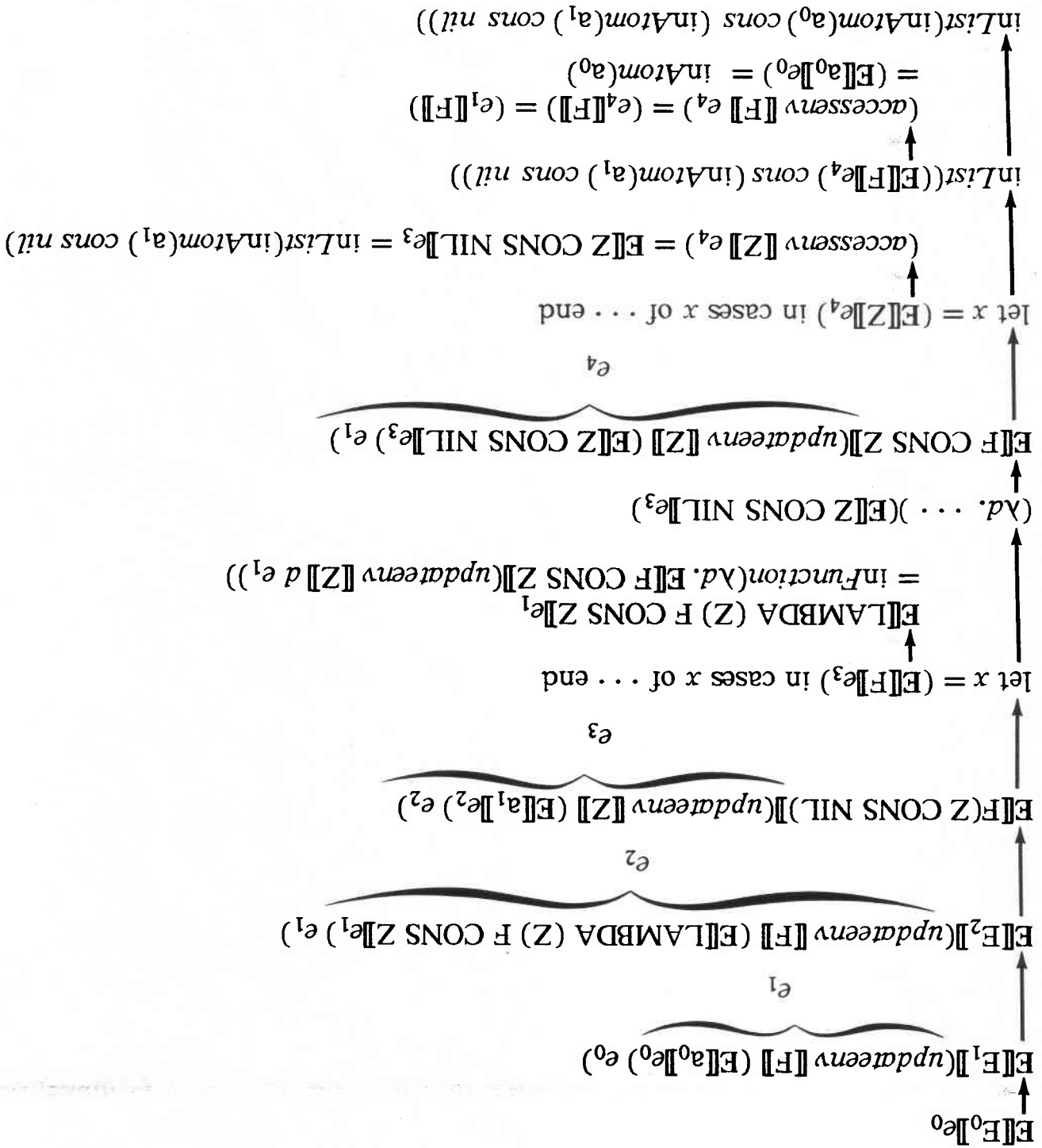
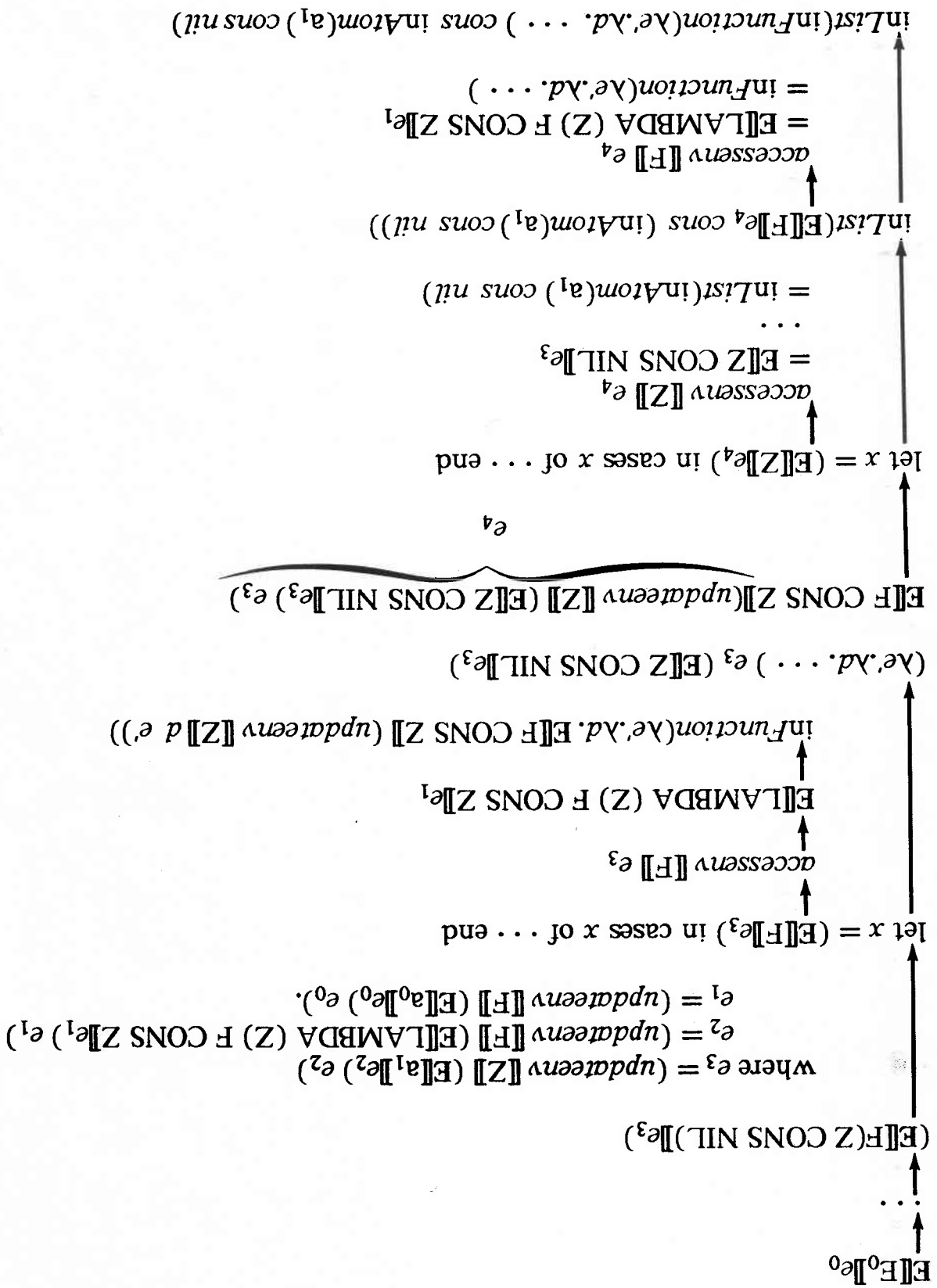


Figure 7.6

Figure 7.7



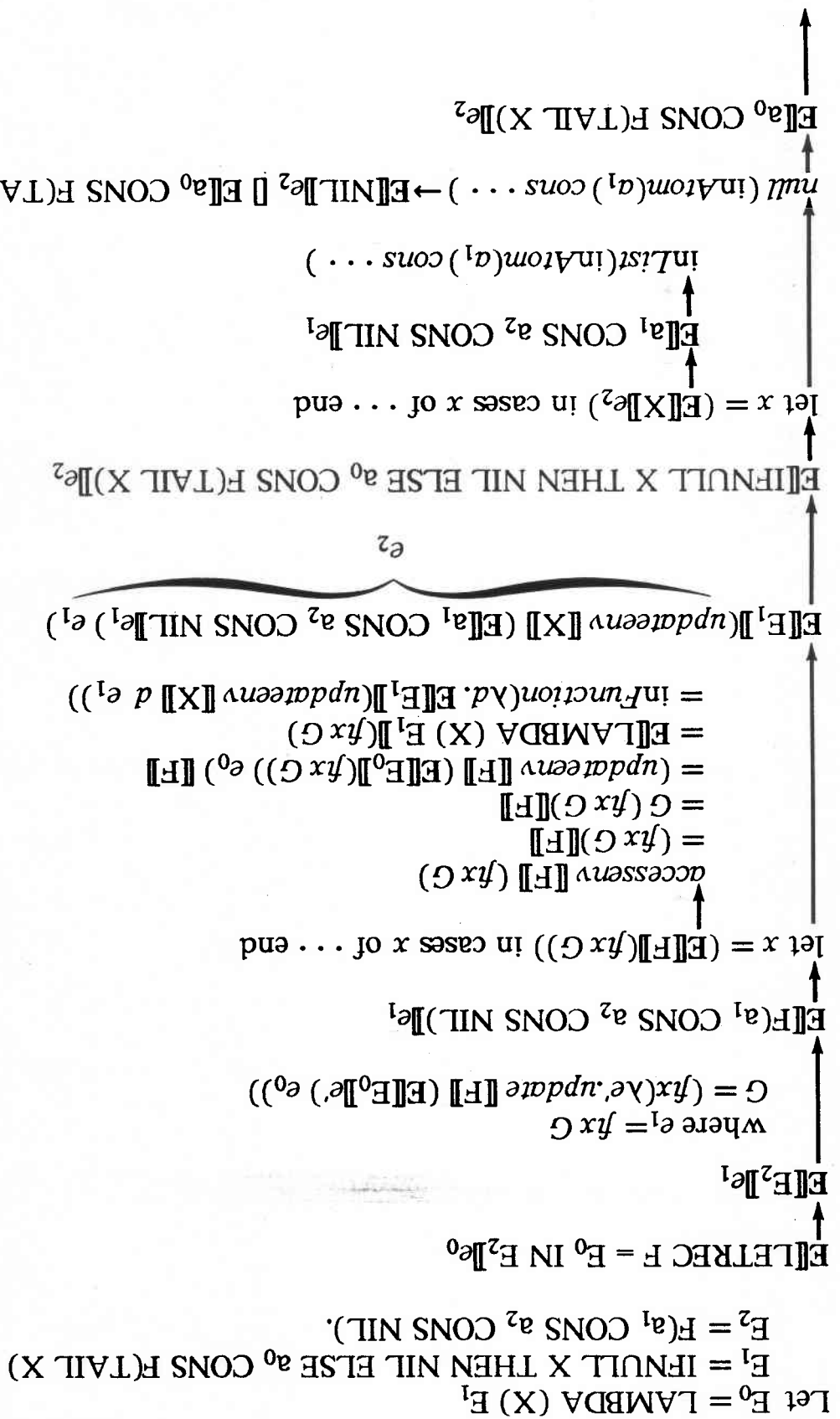


Figure 7.8

Figure 7.8 (continued)

