Problems Solved:

11	12	13	14	15

Name:

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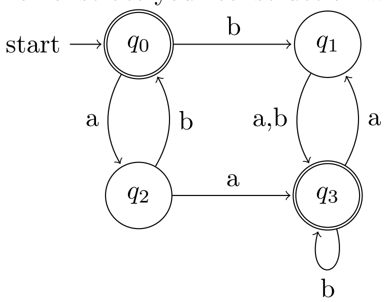
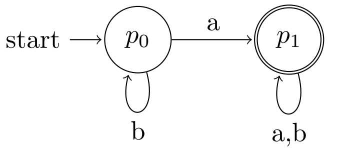
Problem 11. Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be two DFSM over the alphabet Σ. Let $L(M_1)$ and $L(M_2)$ be the languages accepted by M_1 and M_2, respectively.
Construct a DFSM $M = (Q, \Sigma, \delta, q, F)$ whose language $L(M)$ is the intersection of $L(M_1)$ and $L(M_2)$. Write down Q, δ, q, and F explicitly.
Hint: M simulates the parallel execution of M_1 and M_2. For that to work, M "remembers" in its state the state M_1 as well as the state of M_2. This can be achieved by defining $Q = Q_1 \times Q_2$.
Demonstrate your construction with the following DFSMs.



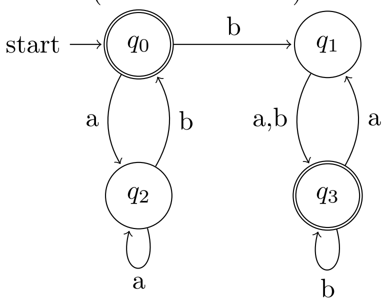


Problem 12. Let $\ominus$ be defined over natural numbers as follows.

$$x \ominus y = \begin{cases} x - y, & \text{if } x \geq y \\ 0, & \text{otherwise} \end{cases}$$

Show that the language $L = \{a^m b^n c^{n \ominus m} \mid m, n \in \mathbb{N}\}$ over the alphabet $\Sigma = \{a, b, c\}$ is not regular.

Problem 13. Let M_1 be the DFSM with states $\{q_1, q_2, q_3, q_4\}$ whose transition graph is given below. Determine a regular expression r such that $L(r) = L(M_1)$. Show the *derivation* of the the final result by the technique based on Arden's Lemma (see lecture notes).



Problem 14. Let r be the following regular expression.

$$(ab + ba)^* + bb$$

Construct a nondeterministic finite state machine N such that $L(N) = L(r)$. Show the derivation of the result by following the technique presented in the proof of the theorem *Equivalence of Regular Expressions and Automata* (see lecture notes).

Problem 15. Let M be the following Turing maschine:

$$\begin{aligned} Q &= \{q_0, q_1, f, r, \lambda, \rho, \tau\} \\ \Sigma &= \{0, 1\} \\ \Gamma &= \{0, 1, \sqcup, X\} \\ F &= \{q_1\} \end{aligned}$$

$$\delta\colon Q \times \Gamma \to Q \times \Gamma \times \{L, S, R\}$$

$$\begin{gathered} \delta(q_0, 1) = \delta(r, 1) = (r, 1, R) \\ \delta(q_0, 0) = \delta(r, 0) = (\tau, X, R) \\ \delta(\tau, 1) = (\lambda, X, S) \\ \delta(\lambda, 1) = \delta(\rho, X) = (\rho, X, R) \\ \delta(\lambda, X) = \delta(\rho, 1) = (\lambda, X, L) \\ \delta(f, \sqcup) = (q_1, \sqcup, S) \\ \delta(f, X) = (f, X, R) \\ \delta(\lambda, \sqcup) = (f, \sqcup, R) \end{gathered}$$

(a) Show the moves (sequences of configurations) performed by the machine for inputs 101 and 1011.

(b) Describe the language $L(M)$ accepted by M as precisely as possible.

Note that the above definition extends the standard definition of a Turing maschine by an additional direction S that denotes “standstill”. When solving the problem you are allowed to make corresponding moves (sequences of configurations) that do not change the tape position.
Remark: Any extendend Turing maschine M with directions L,R,S can be replaced by an equivalent Turing maschine M' with directions L,R by introducing for every transition $\delta(q, x) = (q', x', S)$ of M in M' a new state q'' with

$$\begin{gathered} \delta(q, x) = (q'', x', L) \\ \delta(q'', x) = (q', x, R) \end{gathered}$$

i.e. M rather than standing still moves first one position left and then one position right.