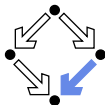


Formal Specification of Abstract Datatypes

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Datatypes



What is a datatype?

- **Traditional view:** collection of data with same structure.

- Mathematics:

$$\text{set } S := \text{int} \times \text{char} = \{(a, b) \mid a \in \text{int} \wedge b \in \text{char}\}.$$

- Programming:

```
struct S {int a; char b}
```

- **Modern view:** collection of data with same services.

- Mathematics

$$\begin{aligned} \text{algebra } T &= (S, \text{getA} : S \rightarrow \text{int}, \text{getB} : S \rightarrow \text{char}) \\ &= (\text{int} \times \text{char}, \lambda(a, b).a, \lambda(a, b).b). \end{aligned}$$

- Programming:

```
class T { S x;  
    int getA() {return x.a}; char getB() {return x.b} }.
```

In this course, we will take the modern view of datatypes.

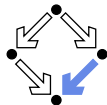


Abstract Datatypes

What is an abstract datatype (ADT)?

- The set of services to be provided by an implementing datatype.
 - The description of the services is the **specification** of the ADT.
 - The specification does not enforce a particular data representation.
 - A datatype providing such services is an **implementation** of the ADT.
 - Provides concrete data representations for the values of the ADT.
 - Provides concrete program methods for the services of the ADT.
- There may be zero, one, **many implementations** of an ADT possible.
 - The specification of the ADT should be as general as possible in order not to constrain the implementation more than necessary.
- The specification is the **contract** between user and implementer.
 - “Design by contract” (Bertrand Meyer).

Thus we need specification languages to describe ADTs.



The screenshot shows a web browser window displaying the Java API documentation for the `java.util.Stack` class. The browser's address bar shows the URL `http://java.sun.com/j2se/1.4.2/docs/api/ja` and the search bar contains `java.util.stack`. The page title is `java.util` and the main heading is `Class Stack`. The navigation menu includes `Overview`, `Package`, `Class` (selected), `Use`, `Tree`, `Deprecated`, `Index`, and `Help`. Below the navigation menu, there are links for `PREV CLASS`, `NEXT CLASS`, `FRAMES`, `NO FRAMES`, and `All Classes`. The `SUMMARY` section lists `FIELD`, `CONSTR`, and `METHOD`. The `DETAIL` section also lists `FIELD`, `CONSTR`, and `METHOD`. The `java.util` package is expanded to show the `Class Stack`. The `Class Stack` section shows the class hierarchy: `java.lang.Object` (parent), `java.util.AbstractCollection` (child), `java.util.AbstractList` (child), `java.util.Vector` (child), and `java.util.Stack` (child). The `All Implemented Interfaces:` section lists `Cloneable`, `Collection`, `List`, `RandomAccess`, and `Serializable`. The `public class Stack` section shows that `Stack` extends `Vector`. The `Stack` class is described as a last-in-first-out (LIFO) stack of objects that extends `Vector` with five operations: `push`, `pop`, `peek`, `empty`, and `search`. The `Stack` class is noted to contain no items when first created.

java.util

Class Stack

[java.lang.Object](#)

- [java.util.AbstractCollection](#)
 - [java.util.AbstractList](#)
 - [java.util.Vector](#)
 - [java.util.Stack](#)

All Implemented Interfaces:
[Cloneable](#), [Collection](#), [List](#), [RandomAccess](#), [Serializable](#)

public class **Stack**
extends [Vector](#)

The stack class represents a last-in-first-out (LIFO) stack of objects. It extends class `Vector` with five operations that allow a vector to be treated as a stack. The usual `push` and `pop` operations are provided, as well as a method to `peek` at the top item on the stack, a method to test for whether the stack is `empty`, and a method to `search` the stack for an item and discover how far it is from the top.

When a stack is first created, it contains no items.



```
public Object push(Object item)
```

Pushes an item onto the top of this stack.

Parameters:

item - the item to be pushed onto this stack.

Returns:

the item argument.

```
public Object pop()
```

Removes the object at the top of this stack and returns that object as the value of this function.

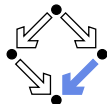
Returns:

The object at the top of this stack.

Throws:

EmptyStackException - if this stack is empty.

Java Interfaces



```
interface StackADT
{
    // Pushes an item onto the top of this stack.
    // Returns the item pushed on the stack.
    Object push(Object item);

    // Removes the object at the top of this stack and
    // returns that object as the value of this function.
    // Throws EmptyStackException, if this stack is empty.
    Object pop();

    // Returns the object at the top of this stack
    // without removing it from the stack.
    // Throws EmptyStackException, if this stack is empty.
    Object peek();

    // Returns true if and only if this stack contains no items.
    boolean empty();
}
```

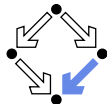


Specification Languages

Programming languages only describe the syntax (interface) of an ADT.

- **Specification languages** also describe the semantics (behavior).
 - Based on concepts from universal algebra and logic.
 - Notions “datatype” and “ADT” have a precise meaning.
 - An algebra T and a (particular) class \mathcal{A} of algebras, respectively.
 - Statement “datatype T implements ADT \mathcal{A} ” has a precise meaning.
 - $T \in \mathcal{A}$.
 - Formal calculus to prove the statement.
- **Constructive specifications** may be even executed.
 - Describe not only requirements but also suggest an implementation.
 - Term rewriting engines for executing constructive specifications.
 - **Rapid prototyping** of specifications in the design phase.

Formal specifications can overcome the ambiguity of natural language when describing program requirements.



```
Stack (E, C): trait
  introduces
    empty: -> C
    push: E, C -> C
    top: C -> E
    pop: C -> C
    isEmpty: C -> Bool
  asserts
    C generated by empty, push
  forall e: E, stk: C
    top(push(e, stk)) == e;
    pop(push(e, stk)) == stk;
    isEmpty(stk) == stk = empty
```

Formal description of ADT “Stack” in the Larch Shared Language (LSL).



```
template <class Elem
/*@ expects contained_objects(Elem) @*/ >
class Stack {
public:

/*@ uses Stack(Elem for E,
        Stack<Elem> for C);

Stack() throw();
/*@ behavior {
/*@ modifies self;
/*@ ensures liberally self' = empty; }

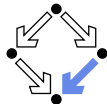
virtual Stack<Elem>& push(Elem e) throw();
/*@ behavior {
/*@ modifies self;
/*@ ensures liberally self' =
/*@   push(self^,e) /\ result = self; }

virtual Stack<Elem>& pop() throw();
/*@ behavior {
/*@ requires ~isEmpty(self^);
/*@ modifies self;
/*@ ensures self' =
/*@   pop(self^) /\ result = self; }

virtual Elem top() const throw();
/*@ behavior {
/*@ requires ~isEmpty(self\any);
/*@ ensures result = top(self\any); }

virtual bool isEmpty() const throw();
/*@ behavior {
/*@ ensures result =
/*@   (isEmpty(self\any)); }
};
```

Formal specification of a C++ “Stack” in Larch/C++.



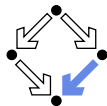
```
> cafeobj

-- loading standard prelude

-- CafeOBJ system Version 1.5.5(PigNose0.99) --
    built: 2015 Dec 28 Mon 1:43:14 GMT
    prelude file: std.bin
    ***
    2016 Feb 2 Tue 9:26:27 GMT
    Type ? for help
    ***
-- Containing PigNose Extensions --
    ---
    built on SBCL
    1.3.1

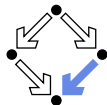
CafeOBJ>
```

System for executing constructive specifications.



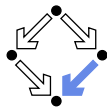
```
CafeOBJ> module! STACK
{
  protecting (NAT)
  signature
  {
    [ Stack ]
    op empty : -> Stack
    op push : Nat Stack -> Stack
    op top : Stack -> Nat
    op pop : Stack -> Stack
  }
  axioms
  {
    var N : Nat
    var S : Stack

    eq top(push(N, S)) = N .
    eq pop(push(N, S)) = S .
  }
}
```



```
CafeOBJ> open STACK .
-- opening module STACK.. done.
%STACK> parse top(push(1, empty)) .
top(push(1,empty)) : Nat
%STACK> reduce top(push(1, empty)) .
-- reduce in %STACK : top(push(1,empty))
1 : NzNat
(0.000 sec for parse, 1 rewrites(0.000 sec), 1 matches)
%STACK> parse top(pop(push(2, push(1, empty)))) .
top(pop(push(2,push(1,empty)))) : Nat
%STACK> reduce top(pop(push(2, push(1, empty)))) .
-- reduce in %STACK : top(pop(push(2,push(1,empty))))
1 : NzNat
(0.000 sec for parse, 2 rewrites(0.000 sec), 2 matches)
%STACK> parse top(pop(push(1, empty))) .
top(pop(push(1,empty))) : Nat
%STACK> reduce top(pop(push(1, empty))) .
-- reduce in %STACK : top(pop(push(1,empty)))
top(empty) : Nat
(0.000 sec for parse, 1 rewrites(0.000 sec), 2 matches)
%STACK> close .
```

Algebraic/Axiomatic Specifications



- Approach rooted in universal algebra.
 - Logical **axioms** relate different operations of ADT to each other.
 - Similar as in the description of **algebras** in mathematics.
- Original focus (1970s/1980s): **initial semantics**.
 - Specifications in (conditional) equational logic.
 - Main interest in executable design specifications.
 - Strong connections to term rewriting.
 - Languages: Clear, ACT ONE/TWO, OBJ family, ...
- Alternative focus (1990s): **loose semantics**.
 - Specifications in full first-order predicate logic.
 - Main interest in precise requirement specifications.
 - Strong connections to object-oriented program specification.
 - Languages: Larch/C++, Java Modeling Language (JML), ...
- **Common Algebraic Specification Language (CASL)**
 - Result of Common Framework Initiative (CoFI), since 1995.
 - Unifying framework for algebraic specifications in different logics.

Course Outline



- Abstract Datatypes.
- *CafeOBJ*.
- Logic.
- Loose Specifications.
- *Larch/C++*, *JML*.
- Term Algebras.
- Initial Specifications.
- Specifications in the Large.
- *CASL*.

Interspersed with presentations of various case studies; exercises both theoretical (paper and pencil) and practical (*CafeOBJ*).