**Problems Solved:**

| 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|

**Name:**

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**Problem 26.** Let $Q(x) = \{y \in \mathbb{N} \mid x \leq y^2\} \subseteq \mathbb{N}$ and $f : \mathbb{N} \to \mathbb{N}$ be the (partial) function

$$f(x) = \begin{cases} \min Q(x) & \text{if } Q(x) \neq \emptyset, \\ \text{undefined} & \text{otherwise.} \end{cases}$$

1. Is $f$ loop computable?
2. Is $f$ a primitive recursive function?
3. Is $f$ a while computable function?
4. Is $f$ a $\mu$-recursive function?

In each case justify your answer. If it is *yes*, give a corresponding program and/or an explicit definition as (primitive/$\mu$-) recursive function.
Remark: When defining $f$, you are allowed to use the Definition 29 and 30 from the lecture notes and the primitive recursive functions (respectively loop programs computing these functions)

$$m : \mathbb{N}^2 \to \mathbb{N}, \quad (x, y) \mapsto x \cdot y$$

and $u : \mathbb{N}^2 \to \mathbb{N}$,

$$u(x, y) = \begin{cases} 0 & \text{if } x = y, \\ 1 & \text{if } x \neq y. \end{cases}$$

Other functions or rules are forbidden.

**Problem 27.** Configurations of a Turing machine $(Q, \Gamma, \sqcup, \Sigma, \delta, q_0, F)$ can be encoded as a terms in various ways; for instance we can encode the configuration

```
                    +-----+
                    |     |
                    |  q  |
                    |     |
                    +-----+
                       |
                       |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+----
|     |     |     |     |     |     |     |     |     |
| x_m | ... | x_2 | x_1 |  z  | y_1 | y_2 | ... | y_n | ...
|     |     |     |     |     |     |     |     |     |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+----
```

as the term

$$g(q, z, f(x_1, f(x_2 \cdots f(x_m, e))), f(y_1, f(y_2 \cdots f(y_m, e)))).$$

In the picture, $q$ is the state of the head and the symbols $x_m, \ldots, x_1, z, y_1, \ldots, y_n \in \Gamma$ describes the tape to the left / under / to the right of the head.
Show how to translate the transition function $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ to a set of term rewrite rules.

1. Give a rewrite rule for each $q \in Q$ and each $c \in \Gamma$ with $\delta(q, c) = (q', c', L)$
2. Give a rewrite rule for each $q \in Q$ and each $c \in \Gamma$ with $\delta(q, c) = (q', c', R)$

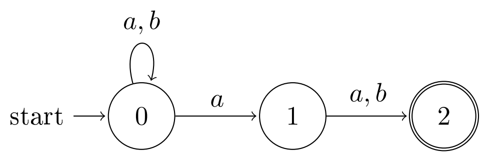
*Hint:* It helps to draw pictures of the machine configuration before and after a transition and to translate both configurations to terms.

**Problem 28.**

(a) The NFSM

$$A = (Q, \Sigma, \delta, Q_0, F)$$

over the alphabet $\Sigma = \{a, b\}$ with the states $Q = \{0, 1, 2\}$, and the starting states $Q_0 = \{0\}$ and accepting states $F = \{2\}$ is given by the following picture.




Give a right-linear grammer $G = (N, \Sigma, P, S)$ with $L(G) = L(A)$ and give a derivation for the sentence *bab*.

(b) Now, let $A = (Q, \Sigma, \delta, Q_0, F)$ be an *arbitrary* NFSM. Give a right linear grammar $G = (N, \Sigma, P, S)$ with $L(G) = L(A)$.

**Problem 29.** Consider the grammar $G = (N, \Sigma, P, S)$ where $N = \{S\}$, $\Sigma = \{a, b\}$, $P = \{S \to \varepsilon, S \to aSbS\}$.

(a) Is $aababb \in L(G)$?

(b) Is $aabab \in L(G)$?

(c) Does every element of $L(G)$ contain the same number of occurrences of $a$ and $b$?

(d) Is $L(G)$ regular?

(e) Is $L(G)$ recursive?

Justify your answers.

**Problem 30.** Construct a DFSM recognizing $L(G)$ where $G = (\{A, B\}, \{a, b\}, P, A)$ with the production rules $P$ given by

$$A \to aA|bB|b,$$
$$B \to aA|aB.$$

*Hint:* Start by a constructing a NFSM $N$. Then turn $N$ into a DFSM $D$ sucht that $L(G) = L(N) = L(D)$.
"Construct" means to explain how you turn the grammar into a DFSM. Simply writing down a DFSM $D$ with the required property, does not count as a solution unless you *prove* that $L(G) = L(D)$.