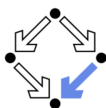
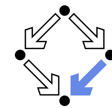
# Specifying and Verifying Programs (Part 2)

Wolfgang Schreiner
Wolfgang.Schreiner@risc.jku.at

Research Institute for Symbolic Computation (RISC)
Johannes Kepler University, Linz, Austria
http://www.risc.jku.at





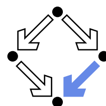


1. Programs as State Relations

2. The RISC ProgramExplorer



## Specification by State Predicates



- Hoare calculus and predicate transformers use state predicates.
  - Formulas that talk about a single (pre/post-)state.
  - In such a formula, a reference $x$ means "the value of program variable $x$ in the given state".
- Relationship between pre/post-state is not directly expressible.
  - Requires uninterpreted mathematical constants.

  $\{x = a\} x := x + 1 \{x = a + 1\}$
- Unchanged variables have to be explicitly specified.

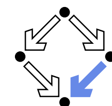
  $\{x = a \wedge y = b\} x := x + 1 \{x = a + 1 \wedge y = b\}$
- The semantics of a command $c$ is only implicitly specified.
  - Specifications depend on auxiliary state conditions $P, Q$.

  $\{P\} c \{Q\}$
  $\text{wp}(c, Q) = P$

Let us turn our focus from individual states to pairs of states.



## Specification by State Relations



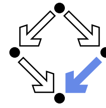
- We introduce formulas that denote state relations.
  - Talk about a pair of states (the pre-state and the post-state).
  - old $x$: "the value of program variable $x$ in the pre-state".
  - var $x$: "the value of program variable $x$ in the post-state".
- We introduce the logical judgment $c : [F]^{x,\ldots}$
  - If the execution of $c$ terminates normally, the resulting post-state is related to the pre-state as described by $F$.
  - Every variable $y$ not listed in the set of variables $x, \ldots$ has the same value in the pre-state and in the post-state.

    $c : F \wedge \text{var } y = \text{old } y \wedge \ldots$

$x := x + 1 : [\text{var } x = \text{old } x + 1]^x$
$x := x + 1 : \text{var } x = \text{old } x + 1 \wedge \text{var } y = \text{old } y \wedge \text{var } z = \text{old } z \wedge \ldots$

We will discuss the termination of commands later.

## State Relation Rules



$$\frac{c : [F]^{xs} \qquad y \notin xs}{c : [F \land \text{var } y = \text{old } y]^{xs \cup \{y\}}}$$

$$\textbf{skip} : [\text{true}]^{\emptyset} \qquad \textbf{abort} : [\text{true}]^{\emptyset} \qquad x = e : [\text{var } x = e']^{x}$$

$$\frac{c_1 : [F_1]^{xs} \quad c_2 : [F_2]^{xs}}{c_1; c_2 : [\exists ys : F_1[ys/\text{var } xs] \land F_2[ys/\text{old } xs]]^{xs}}$$

$$\frac{c : [F]^{xs}}{\textbf{if } e \textbf{ then } c : [\text{if } e' \text{ then } F \text{ else var } xs = \text{old } xs]^{xs}}$$

$$\frac{c_1 : [F_1]^{xs} \qquad c_2 : [F_2]^{xs}}{\textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 : [\text{if } e' \text{ then } F_1 \text{ else } F_2]^{xs}}$$

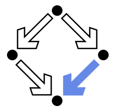
$$\frac{\begin{array}{c} c : [F]^{xs} \\ \vdash \forall xs, ys, zs : I[xs/\text{old } xs, ys/\text{var } xs] \land e[ys/xs] \land F[ys/\text{old } xs, zs/\text{var } xs] \Rightarrow \\ I[xs/\text{old } xs, zs/\text{var } xs] \end{array}}{\textbf{while } e \textbf{ do } \{I, t\} \ c : [\neg e'' \land (I[\text{old } xs/\text{var } xs] \Rightarrow I)]^{xs}}$$

if $e$ then $F_1$ else $F_2$ $:\Leftrightarrow (e \Rightarrow F_1) \land (\neg e \Rightarrow F_2)$

$e' := e[\text{old } xs/xs]$, $e'' := e[\text{var } xs/xs]$ (for all program variables $xs$)



## Example



$c_1 = y := y + 1;$
$c_2 = x := x + y$

$c_1 : [\text{var } y = \text{old } y + 1]^{y}$
$c_2 : [\text{var } x = \text{old } x + \text{old } y]^{x}$

$c_1 : [\text{var } y = \text{old } y + 1 \land \text{var } x = \text{old } x]^{x,y}$
$c_2 : [\text{var } x = \text{old } x + \text{old } y \land \text{var } y = \text{old } y]^{x,y}$

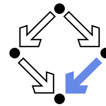
$c_1; c_2 : [\exists x_0, y_0 :$
$\quad y_0 = \text{old } y + 1 \land x_0 = \text{old } x \land$
$\quad \text{var } x = x_0 + y_0 \land \text{var } y = y_0]^{x,y}$

$c_1; c_2 : [\text{var } x = \text{old } x + \text{old } y + 1 \land \text{var } y = \text{old } y + 1]^{x,y}$

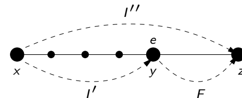
Mechanical translation and logical simplification.



## Loops



$$\frac{\begin{array}{c} c : [F]^{xs} \\ \vdash \forall xs, ys, zs : I[xs/\text{old } xs, ys/\text{var } xs] \land e[ys/xs] \land F[ys/\text{old } xs, zs/\text{var } xs] \Rightarrow \\ I[xs/\text{old } xs, zs/\text{var } xs] \end{array}}{\textbf{while } e \textbf{ do } \{I, t\} \ c : [\neg e'' \land (I[\text{old } xs/\text{var } xs] \Rightarrow I)]^{xs}}$$



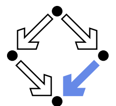

$w = \textbf{while } i < n \textbf{ do } \{I, t\} \ (s := s + i; i := i + 1)$
$I \Leftrightarrow 0 \leq \text{var } i \leq \text{old } n \land \text{var } s = \sum_{j=0}^{\text{var } i-1} j$

$$\frac{\begin{array}{l} (s := s + i; i := i + 1) : [\text{var } s = \text{old } s + \text{old } i \land \text{var } i = \text{old } i + 1]^{s,i} \\ \vdash \forall s_x, s_y, s_z, i_x, i_y, i_z : \\ \quad (0 \leq i_y \leq \text{old } n \land s_y = \sum_{j=0}^{i_y - 1} j) \land i_y < \text{old } n \land (s_z = s_y + i_y \land i_z = i_y + 1) \Rightarrow \\ \quad\quad 0 \leq i_z \leq \text{old } n \land s_z = \sum_{j=0}^{i_z - 1} j \end{array}}{w : [\neg(\text{var } i < \text{var } n) \land (0 \leq \text{old } i \leq \text{old } n \land \text{old } s = \sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}}$$

The loop relation is derived from the invariant (not the loop body); we have to prove the preservation of the loop invariant.



## Example



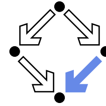
$c =$
$\quad \textbf{if } n < 0$
$\quad\quad s := -1$
$\quad \textbf{else}$
$\quad\quad s := 0$
$\quad\quad i := 0$
$\quad\quad \textbf{while } i < n \textbf{ do } \{\text{I,t}\}$
$\quad\quad\quad s := s + i$
$\quad\quad\quad i := i + 1$

$I \Leftrightarrow 0 \leq \text{var } i \leq \text{old } n \land \text{var } s = \sum_{j=0}^{\text{var } i-1} j$
$t = \text{old } n - \text{old } i$

$c : [\text{if old } n < 0$
$\quad \text{then var } i = \text{old } i \land \text{var } s = -1$
$\quad \text{else var } i = \text{old } n \land \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$

Let us calculate this “semantic essence” of the program.

## Example



$c = \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b$
$b = (s := 0; i := 0; w)$
$w = \textbf{while } i < n \textbf{ do } \{I, t\}\ (s := s + i; i = i + 1)$

$s := 0 : [\text{var } s = 0]^{s}$
$s := 0 : [\text{var } s = 0 \wedge \text{var } i = \text{old } i]^{s,i}$

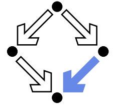
$i := 0 : [\text{var } i = 0]^{i}$
$i := 0 : [\text{var } i = 0 \wedge \text{var } s = \text{old } s]^{s,i}$

$s := 0; i := 0 : [\exists s_0, i_0 : s_0 = 0 \wedge i_0 = \text{old } i \wedge\ \text{var } i = 0 \wedge \text{var } s = s_0]^{s,i}$
$s := 0; i := 0 : [\text{var } s = 0 \wedge \text{var } i = 0]^{s,i}$

$w : [\neg(\text{var } i < \text{var } n) \wedge (0 \leq \text{old } i \leq \text{old } n \wedge \text{old } s = \sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}$
$w : [\text{var } i \geq \text{old } n \wedge (0 \leq \text{old } i \leq \text{old } n \wedge \text{old } s = \sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}$



## Example



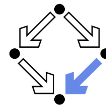
$c = \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b$
$b = (s := 0; i := 0; w)$
$w = \textbf{while } i < n \textbf{ do } \{I, t\}\ (s := s + i; i = i + 1)$

$s := 0; i := 0 : [\text{var } s = 0 \wedge \text{var } i = 0]^{s,i}$
$w : [\text{var } i \geq \text{old } n \wedge (0 \leq \text{old } i \leq \text{old } n \wedge \text{old } s = \sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}$

$b : [\exists s_0, i_0 : s_0 = 0 \wedge i_0 = 0 \wedge$
$\quad \text{var } i \geq \text{old } n \wedge (0 \leq i_0 \leq \text{old } n \wedge s_0 = \sum_{j=0}^{i_0-1} j \Rightarrow I)]^{s,i}$
$b : [\exists s_0, i_0 : s_0 = 0 \wedge i_0 = 0\ \wedge$
$\quad \text{var } i \geq \text{old } n \wedge (0 \leq \text{old } n \Rightarrow I)]^{s,i}$
$b : [\text{var } i \geq \text{old } n\ \wedge$
$\quad (0 \leq \text{old } n \Rightarrow 0 \leq \text{var } i \leq \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{var } i-1} j)]^{s,i}$
$b : [\text{var } i \geq \text{old } n\ \wedge$
$\quad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$



## Example



$c = \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b$
$b = (s := 0; i := 0; w)$
$w = \textbf{while } i < n \textbf{ do } \{I, t\}\ (s := s + i; i = i + 1)$

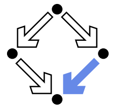
$s := -1 : [\text{var } s = -1]^{s}$
$s := -1 : [\text{var } i = \text{old } i \wedge \text{var } s = -1]^{s,i}$

$b : [\text{var } i \geq \text{old } n\ \wedge$
$\quad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$

$c : [\text{if old } n < 0$
$\quad \text{then var } i = \text{old } i \wedge \text{var } s = -1$
$\quad \text{else var } i \geq \text{old } n\ \wedge$
$\qquad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$
$c : [\text{if old } n < 0$
$\quad \text{then var } i = \text{old } i \wedge \text{var } s = -1$
$\quad \text{else var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$



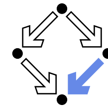
## Partial Correctness



- Specification $(xs, P, Q)$
  - Set of program variables $xs$ (which may be modified).
  - Precondition $P$ (a formula with "old $xs$" but no "var $xs$").
  - Postcondition $Q$ (a formula with both "old $xs$" and "var $xs$").
- Partial correctness of implementation $c$
  1. Derive $c : [F]^{xs}$.
  2. Prove $F \Rightarrow (P \Rightarrow Q)$
     - Or: $P \Rightarrow (F \Rightarrow Q)$
     - Or: $(P \wedge F) \Rightarrow Q$

Verification of partial correctness leads to the proof of an implication.

## Relationship to Other Calculi



Let all state conditions refer via "*old* xs" to program variables *xs*.

- Hoare Calculus
  - For proving $\{P\}c\{Q\}$,
  - it suffices to derive $c : [F]^{xs}$
  - and prove $P \wedge F \Rightarrow Q[\text{var } xs/\text{old } xs]$.
- Predicate Transformers
  - Assume we can derive $c : [F]^{xs}$.
  - If $c$ does not contain loops, then

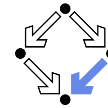
$$\text{wp}(c, Q) = \forall xs : F[xs/\text{var } xs] \Rightarrow Q[xs/\text{old } xs]$$
$$\text{sp}(c, P) = \exists xs : P[xs/\text{old } xs] \wedge F[xs/\text{old } xs, \text{old } xs/\text{var } xs]$$

  - If $c$ contains loops, the result is still a valid pre/post-condition but not necessarily the weakest/strongest one.

A generalization of the previously presented calculi.



## Termination



- We introduce a judgment $c \downarrow T$.

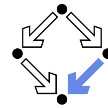
  State condition $T$ (a formula with "old *xs*" but no "var *xs*").
  - Starting with a pre-state that satisfies condition $T$ the execution of command $c$ terminates.
- Total correctness of implementation $c$.

  Specification $(xs, P, Q)$.
  - Derive $c \downarrow T$.
  - Prove $P \Rightarrow T$.

Also verification of termination leads to the proof of an implication.



## Termination Condition Rules



$$\textbf{skip} \downarrow \text{true} \qquad \textbf{abort} \downarrow \text{true} \qquad x := e \downarrow \text{true}$$

$$\frac{c_1 \downarrow T_1 \qquad c_2 \downarrow T_2}{c_1; c_2 \downarrow T_1 \wedge \text{wp}(c_1, T_2)}$$

$$\frac{c \downarrow T}{\textbf{if } e \textbf{ then } c \downarrow e' \Rightarrow T}$$

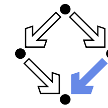
$$\frac{c_1 \downarrow T_1 \qquad c_2 \downarrow T_2}{\textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 \downarrow \text{if } e' \text{ then } T_1 \text{ else } T_2}$$

$$\frac{\begin{array}{c} c : [F]^{xs} \qquad c \downarrow T \\ \vdash \forall xs, ys, zs : I[xs/\text{old } xs, ys/\text{var } xs] \wedge e[ys/xs] \wedge F[ys/\text{old } xs, zs/\text{var } xs] \Rightarrow \\ T[ys/\text{old } xs] \wedge 0 \le t[zs/\text{old } xs] < t[ys/\text{old } xs] \end{array}}{\textbf{while } e \textbf{ do } \{I, t\}\ c \downarrow t \ge 0}$$

In every iteration of a loop, the loop body must terminate and the termination term must decrease (but not become negative).



## Example



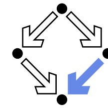
$c =$
  **if** $n < 0$
    $s := -1$
  **else**
    $s := 0$
    $i := 0$
    **while** $i < n$ **do** {I,t}
      $s := s + i$
      $i := i + 1$

$I \Leftrightarrow 0 \le \text{var } i \le \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{var } i-1} j]$
$t = \text{old } n \ - \ \text{old } i$

$c \downarrow \text{if old } n < 0 \text{ then true else } \ldots$
$c \downarrow \text{if old } n < 0 \text{ then true else old } n \ge 0$
$c \downarrow \text{true}$

We still have to prove the constraint on the loop iteration.

## Example



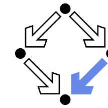
$s := s + i; i := i + 1 \downarrow \text{true}$

$\forall s_x, s_y, s_z, i_x, i_y, i_z :$
$\quad (0 \leq i_y \leq \text{old } n \wedge s_y = \sum_{j=0}^{i_y - 1} j) \wedge$
$\quad i_y < \text{old } n \wedge$
$\quad (s_z = s_y + i_y \wedge i_z = i_y + 1) \Rightarrow$
$\qquad \text{true} \wedge$
$\qquad 0 \leq \text{old } n - i_z < \text{old } n - i_y$

Also this constraint is simple to prove.



## Abortion

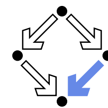


Also abortion can be ruled out by proving side conditions in the usual way.

Wolfgang Schreiner. *Computer-Assisted Program Reasoning Based on a Relational Semantics of Programs.* Research Institute for Symbolic Computation (RISC), Johannes Kepler University, Linz, Austria, 2011.

See the report for the full calculus.



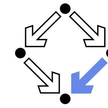


1. Programs as State Relations

2. The RISC ProgramExplorer



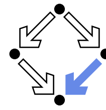
## The RISC ProgramExplorer



- An integrated environment for program reasoning.
  - Research Institute for Symbolic Computation (RISC), 2008–.
    http://www.risc.jku.at/research/formal/software/ProgramExplorer
    - Integrates the RISC ProofNavigator for computer-assisted proving.
    - Written in Java, runs under Linux (only), freely available (GPL).
- Programs written in "MiniJava".
  - Subset of Java with full support of control flow interruptions.
  - Value (not pointer) semantics for arrays and objects.
- Theories and specifications written in a formula language.
  - Derived from the language of the RISC ProofNavigator.
- Semantic analysis and verification.
  - Program methods are translated into their "semantic essence".
    - Open for human inspection.
  - From the semantics, the verification tasks are generated.
    - Solved by automatic decision procedure or interactive proof.

Tight integration of executable programs, declarative specifications, mathematical semantics, and verification tasks.

# Using the Software

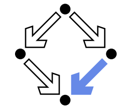


See "The RISC ProgramExplorer: Tutorial and Manual".

- Develop a theory.
  - File "*Theory*.theory" with a `theory` *`Theory`* of mathematical types, constants, functions, predicates, axioms, and theorems.
  - Can be also added to a program file.
- Develop a program.
  - File "*Class*.java" with a `class` *`Class`* that contains class (static) and object (non-static) variables, methods and constructors.
  - Class may be annotated by a theory (and an object invariant).
  - Methods may be annotated by method specifications.
  - Loops may be annotated by invariants and termination terms.
- Analyze method semantics.
  - Transition relations, termination conditions, ... of the method body and its individual commands.
- Perform verification tasks.
  - Frame, postcondition, termination, preconditions, loop-related tasks, type-checking conditions.



# Starting the Software



- Starting the software:

  `module load ProgramExplorer` (only at RISC)
  `ProgramExplorer &`
- Command line options:

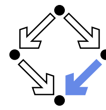
```
Usage: ProgramExplorer [OPTION]...
OPTION: one of the following options:
    -h, --help: print this message.
    -cp, --classpath [PATH]:
      directories representing top package.

Environment Variables:
    PE_CLASSPATH:
        the directories (separated by ":") representing the
        top package (default the current working directory)
    ...
```

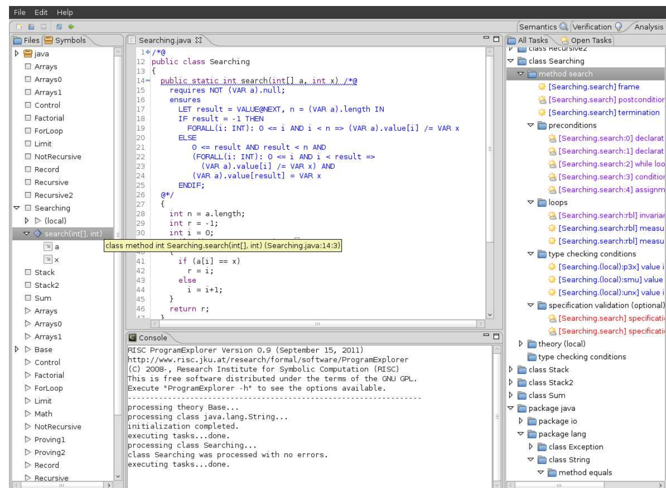
- Task repository created/read in current working directory:

  Subdirectory `.PETASKS.`*`timestamp`* (ProgramExplorer tasks)
  Subdirectory `.ProofNavigator` (ProofNavigator legacy)



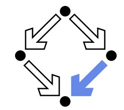
# The Graphical User Interface







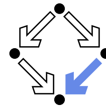
# A Program



```
/*@..
class Sum
{
  static int sum(int n) /*@..
  {
    int s;
    if (n < 0)
      s = -1;
    else
    {
      s = 0;
      int i = 1;
      while (i <= n) /*@..
      {
        s = s+i;
        i = i+1;
      }
    }
    return s;
  }
}
```

Markers `/*@..` indicate hidden mathematical annotations.

## A Theory

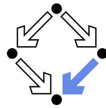


```
/*@
  theory {
    sum: (INT, INT) -> INT;
    sumaxiom: AXIOM
      FORALL(m: INT, n: INT):
        IF n<m THEN
          sum(m, n) = 0
        ELSE
          sum(m, n) = n+sum(m, n-1)
        ENDIF;
  }
@*/
class Sum
...
```

The introduction of a function $sum(m, n) = \sum_{j=m}^{n} j$.



## A Method Specification

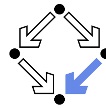


```
static int sum(int n) /*@
  requires VAR n < Base.MAX_INT;
  ensures
    LET result=VALUE@NEXT IN
    IF VAR n < 0
      THEN result = -1
      ELSE result = sum(1, VAR n)
    ENDIF;
@*/
...
```

For non-negative $n$, a call of program method `sum(`$n$`)` returns $sum(1, n)$ (and does not modify any global variable).



## A Loop Annotation

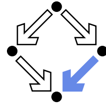


```
while (i <= n) /*@
  invariant VAR n < Base.MAX_INT
        AND 1 <= VAR i AND VAR i <= VAR n+1
        AND VAR s=sum(1, VAR i-1);
  decreases VAR n - VAR i + 1;
@*/
{
  s = s+i;
  i = i+1;
}
}
```

The loop invariant and termination term (measure).



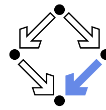
## The Specification Language



Derived from the language of the RISC ProofNavigator.

- State conditions/relations, state terms.
  - State condition: method precondition (`requires`).
  - State relation: method postcondition (`ensures`), loop invariant (`invariant`).
  - State term: termination term (`decreases`).
- References to program variables.
  - `OLD` $x$: the value of program variable $x$ in the pre-state.
  - `VAR` $x$: the value of program variable $x$ in the post-state.
  - In state conditions/terms, both refer to the value in the current state.
  - If program variable is of the program type $T$, then then `OLD`/`VAR` $x$ is of the mathematical type $T'$.
    - `int` $\rightarrow$ `Base.int=[Base.MIN_INT,Base.MAX_INT]`.
- Function results
  - `VALUE@NEXT`: the return value of a program function.
  - The value of the function call's post-state `NEXT`.

## The Semantics View



File Edit Help

Semantics Verification Analysis

Sum.sum

**Sum.sum**

**requires** old $n <$ Base.MAX$_{INT}$
**ensures** let result = value@next
in
( if var $n < 0$ then result $= -1$ else result = sum(1, var $n$) endif )

```
public static int sum(int n) /*@
  requires OLD n < Base.MAX_INT;
  ensures LET result = VALUE@NEXT IN (IF VAR n < 0
@*/
{
  int s;
  if (n < 0)
    s = -1;
  else
  {
    s = 0;
    int i = 1;
    while (i <= n)/*@
      invariant VAR n < Base.MAX_INT AND 1 <= VAR
      decreases OLD n-OLD i+1;
    @*/
    {
      s = s+i;
      i = i+1;
    }
  }
  return s;
}
```

Select a statement and define a condition for its pre/poststate:

Submit Reset Prestate Poststate

**Body Knowledge**

[Show Original Formulas]

**Pre-State Knowledge**

old $n <$ Base.MAX$_{INT}$

**Effects**

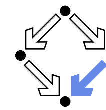
**executes:** false, **continues:** false, **breaks:** false, **returns:** true
**variables:** -; **exceptions:**-

**Transition Relation**

if old $n < 0$ then
returns@next ∧ value@next $= -1$
else
returns@next
∧
($\exists$in∈Base.int: in = old $n+1$ ∧ $1 \leq$ in ∧ value@next = sum(1, in−1))
∧
old $n <$ Base.MAX$_{INT}$
endif



## The Method Body



**Body Knowledge**

[Show Original Formulas]

**Pre-State Knowledge**

old $n <$ Base.MAX$_{INT}$

**Effects**

**executes:** false, **continues:** false, **breaks:** false, **returns:** true
**variables:** -; **exceptions:**-

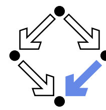
**Transition Relation**

if old $n < 0$ then
returns@next ∧ value@next $= -1$
else
returns@next
∧
($\exists$in∈Base.int: in = old $n+1$ ∧ $1 \leq$ in ∧ value@next = sum(1, in−1))
∧
old $n <$ Base.MAX$_{INT}$
endif

Select method symbol "sum" and menu entry "Show Semantics".



## A Body Command



**Statement Knowledge**

[Show Original Formulas]

**Pre-State Knowledge**

old $n <$ Base.MAX$_{INT}$ ∧ old $n \geq 0$ ∧ old $s = 0$ ∧ old $i = 1$

**Precondition**

old $n <$ Base.MAX$_{INT}$ ∧ $1 \leq$ old $i$ ∧ old $i \leq$ old $n+1$ ∧ old $s$ = sum(1, old $i-1$)

**Effects**

**executes:** true, **continues:** false, **breaks:** false, **returns:** false
**variables:** $s$, $i$; **exceptions:**-

**Transition Relation**

var $i$ = old $n+1$ ∧ old $n <$ Base.MAX$_{INT}$ ∧ $1 \leq$ var $i$ ∧ var $s$ = sum(1, var $i-1$)

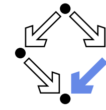
**Termination Condition**

executes@now ⇒ old $n-$old $i \geq -1$

Move the mouse pointer over the box to the left of the loop.



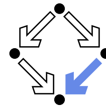
## The Semantics Elements



- Pre-State Knowledge
  What is known about the pre-state of the command.
- Precondition
  What has to be true for the pre-state of the command such that the command may be executed.
- Effects
  Which kind of effects may the command have.
  - variables: which variables may be changed.
  - exceptions: which exceptions may be thrown.
  - executes, continues, breaks, returns: may the execution terminate normally, may it be terminated by a `continue`, `break`, `return`.
- Transition Relation
  The prestate/poststate relationship of the command.
- Termination
  What has to be true for the pre-state of the command such that the command terminates.

Formulas are shown after simplification (see "Show Original Formulas").

## Constraining a State



Select the loop body, enter in the box the condition `VAR s=2 AND VAR i=1`, press "Submit", and move the mouse to `i=i+1`.

**State Conditions**

[Show Original Formulas]

**Pre-State Condition**

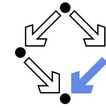
var $i = 1 \wedge$ var $s =$ var $i + 2$

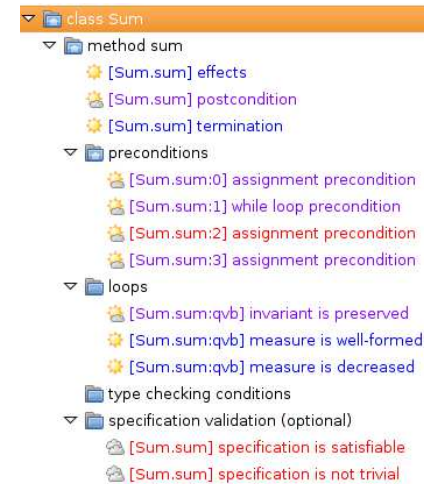
**Post-State Condition**

var $s = 3 \wedge$ var $i = 2$



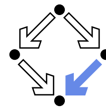
## The Verification Tasks








## The Verification Tasks

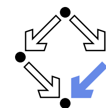


- Effects: does the method only change those global variables indicated in the method's `assignable` clause?
- Postcondition: do the method's precondition and the body's state relation imply the method's postcondition?
- Termination: does the method's precondition imply the body's termination condition?
- Precondition: does a statement's prestate knowledge imply the statement's precondition?
- Loops: is the loop invariant preserved, the measure well-formed (does not become negative) and decreased?
- Type checking conditions: are all formulas well-typed?
- Specification validation: does for every input that satisfies a precondition exist a result that does (not) satisfy the postcondition?

Partially solved by automatic decision procedure, partially by an interactive computer-supported proof.



## The Task States

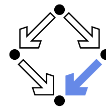


The task status is indicated by color (icon).

- Blue (sun): the task was solved in the current execution of the RISC ProgramExplorer (automatically or by an interactive proof).
- Violet (partially clouded): the task was solved in a previous execution by an interactive proof.
  - Nothing has changed, so we need not perform the proof again.
  - However, we may replay the proof to investigate it.
- Red (partially clouded): there exists a proof but it is either not complete or cannot be trusted any more (something has changed).
- Red (fully clouded): there does not yet exist a proof.

Select "Execute Task" to start/replay a proof, "Show Proof" to display a proof, "Reset Task" to delete a proof.

## A Postcondition Proof



File Options Help

Semantics Verification Analysis

Proof Tree

- [kxy]: scatter
  - [upf]: proved (CVCL)
  - [vpf]: split jm4
    - [rtb]: proved (CVCL)
    - [stb]: scatter
      - [mlv]: proved (CVCL)

Proof State

6zh $\forall m \in \mathbb{Z}, n \in \mathbb{Z}$: if $n < m$ then $sum(m, n) = 0$ else $sum(m, n) = n + sum(m, n-1)$ endif

hfr $n_{old} < MAX_{INT}$
$\wedge$
if $n_{old} < 0$ then
returns_(now_) $\wedge$ value_(now_)+1 = 0
else
returns_(now_)
$\wedge$
($\exists$in$\in$int:in = $n_{old}$+1 $\wedge$ 1 $\leq$ in $\wedge$ value_(now_) = sum(1, in−1))
$\wedge$
$n_{old} < MAX_{INT}$
endif
$\Rightarrow$
if $n_{old} < 0$ then value_(now_)+1 = 0 else value_(now_) = sum(1, $n_{old}$) endif

Children: [upf] [vpf]

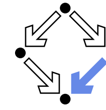
View Declarations

Input/Output

```
Value newArray:[0..MAX_INT]->[#value:ARRAY [MIN_INT..MAX_INT] OF
[#null:BOOLEAN, new:INT#], length:[0..MAX_INT], null:BOOLEAN#].
Value x:[0..MAX_INT].
Value y:[#value:ARRAY [MIN_INT..MAX_INT] OF [#null:BOOLEAN, new:INT#],
length:[0..MAX_INT], null:BOOLEAN#].
Value i:[0..MAX_INT].
Formula goal_ already has a (skeleton) proof (proof status: trusted, closed,
absolute)
Proof state [upf] is closed by decision procedure.
Proof state [rtb] is closed by decision procedure.
Proof state [mlv] is closed by decision procedure.
Proof replay successful.
Use 'proof goal_' to see proof.
```



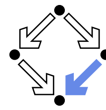
## Linear Search



```
/*@..
public class Searching
{
  public static int search(int[] a, int x) /*@..
  {
    int n = a.length;
    int r = -1;
    int i = 0;
    while (i < n && r == -1) /*@..
    {
      if (a[i] == x)
        r = i;
      else
        i = i+1;
    }
    return r;
  }
}
```



## The Representation of Arrays



The program type `int[]` is mapped to the mathematical type `Base.IntArray`.

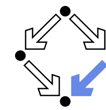
```
theory Base
{
  ...
  IntArray: TYPE =
    [#value: ARRAY int OF int, length: nat, null: BOOLEAN#];
  ...
}
```

- `(VAR a).length`: the number of elements in array *a*.
- `(VAR a).value[i]`: the element with index *i* in array *a*.
- `(VAR a).null`: *a* is the `null` pointer.

Program type *`Class`* is mapped to mathematical type *`Class.Class`*; *`Class`*`[]` is mapped to *`Class`*`.Array`.

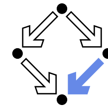


## Theory



```
/*@
theory uses Base {
  int: TYPE = Base.int;
  intArray: TYPE = Base.IntArray;
  smallestPosition: FORMULA
    FORALL(a: intArray, n: NAT, x: int):
      (EXISTS(i:int): 0 <= i AND i < n AND a.value[i] = x) =>
      (EXISTS(i:int): 0 <= i AND i < n AND a.value[i] = x AND
         (FORALL(j:int): 0 <= j AND j < n AND a.value[j] = x =>
           j >= i));
}
@*/
public class Searching
...
```

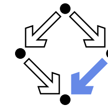
# Method Specification



```
public static int search(int[] a, int x) /*@
  requires (VAR a).null = FALSE;
  ensures
    LET result = VALUE@NEXT, n = (VAR a).length IN
    IF result = -1 THEN
      FORALL(i: INT): 0 <= i AND i < n =>
        (VAR a).value[i] /= VAR x
    ELSE
       0 <= result AND result < n AND
       (FORALL(i: INT): 0 <= i AND i < result =>
         (VAR a).value[i] /= VAR x) AND
       (VAR a).value[result] = VAR x
    ENDIF;
@*/
...
```



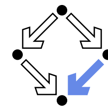
# Loop Annotation



```
while (i < n && r == -1) /*@
  invariant (VAR a).null = FALSE AND VAR n = (VAR a).length
        AND 0 <= VAR i AND VAR i <= VAR n
        AND (FORALL(i: INT): 0 <= i AND i < VAR i =>
              (VAR a).value[i] /= VAR x)
        AND (VAR r = -1 OR (VAR r = VAR i AND VAR i < VAR n AND
               (VAR a).value[VAR r] = VAR x));
  decreases IF VAR r = -1 THEN VAR n - VAR i ELSE 0 ENDIF;
@*/
{
  if (a[i] == x)
    r = i;
  else
    i = i+1;
}
```



# Method Semantics



**Transition Relation**

$(\exists \text{in} \in \text{Base.int}, n \in \text{Base.int}:$

$\quad n = \text{old}\ a.\text{length} \wedge (\text{in} \geq n \vee \text{value@next} \neq -1) \wedge 0 \leq \text{in} \wedge \text{in} \leq n$

$\quad \wedge$

$\quad (\forall i \in \mathbb{Z}: 0 \leq i \wedge i < \text{in} \Rightarrow \text{old}\ a.\text{value}[i] \neq \text{old}\ x)$

$\quad \wedge$

$\quad (\ \text{value@next} = -1$

$\quad\quad \vee$

$\quad\quad \text{value@next} = \text{in} \wedge \text{in} < n \wedge \text{old}\ a.\text{value}[\text{value@next}] = \text{old}\ x)) \wedge \neg\, \text{old}\ a.\text{null}$

$\wedge$

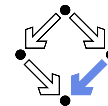
$\text{returns@next}$

**Termination Condition**

$\text{executes@now} \Rightarrow \text{old}\ a.\text{length} \geq 0$

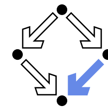


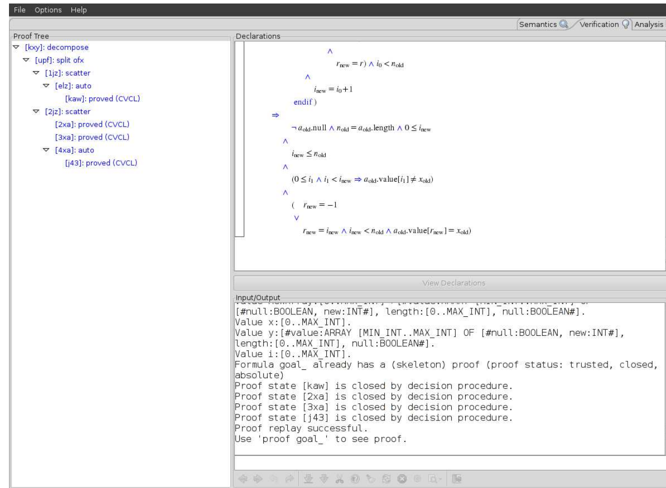
# Verification Tasks



- method search
  - [Searching.search] effects
  - [Searching.search] postcondition
  - [Searching.search] termination
  - preconditions
    - [Searching.search:0] declaration precondition
    - [Searching.search:1] declaration precondition
    - [Searching.search:2] while loop precondition
    - [Searching.search:3] conditional precondition
    - [Searching.search:4] assignment precondition
  - loops
    - [Searching.search:rbl] invariant is preserved
    - [Searching.search:rbl] measure is well-formed
    - [Searching.search:rbl] measure is decreased
  - type checking conditions
    - [Searching.(local):p3x] value is in interval
    - [Searching.(local):smu] value is in interval
    - [Searching.(local):unx] value is in interval
  - specification validation (optional)

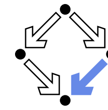
# Invariant Proof








# Working Strategy



- Develop theory.
  - Introduce interesting theorems that may be used in verifications.
- Develop specifications.
  - Validate specifications, e.g. by showing satisfiability and non-triviality.
- Develop program with annotations.
  - Validate programs/annotations by investigating program semantics.
- Prove postcondition and termination.
  - Partial and total correctness.
  - By proofs necessity of additional theorems may be detected.
- Prove precondition tasks and loop tasks.
  - By proofs necessity of additional theorems may be detected.
- Prove mathematical theorems.
  - Validation of auxiliary knowledge used in verifications.

The integrated development of theories, specifications, programs, annotations is crucial for the design of provably correct programs.