

Formal Specification of Abstract Datatypes

Exercise 3 (May 26)

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The result for each exercise is to be submitted by the deadline stated above via the Moodle interface as a .zip or .tgz file which contains

- a PDF file with
 - a cover page with the title of the course, your name, Matrikelnummer, and email-address,
 - the content required by the exercise (specification, source, proof),
- (if required) the CafeOBJ (.mod) file(s) of the specifications.

Exercise 3: Strict Adequacy of Specification

Give a *strictly* adequate specification of the integers (see Exercise 2) and prove that it is indeed strictly adequate w.r.t the “classical” integer algebra with carrier \mathbb{Z} . You only need to show the homomorphism condition for the operations 0_i , 1_i , $+_i$, and $=_i$.

Hint: As shown in class, it is easiest to perform the proof, if you have a unique term representation for every element of \mathbb{Z} . You may therefore base this exercise on Solution 1 of Exercise 2 which already provides a unique term representation.

Bonus (30%): alternatively you may base this exercise also on Solution 2 by replacing the interpretation $\mathcal{M}(sp)(int) = \mathbb{N} \times \mathbb{N}$ with the interpretation

$$\begin{aligned}\mathcal{M}(sp)(int) &= [\mathbb{N} \times \mathbb{N}] / \sim \\ (m_1, n_1) \sim (m_2, n_2) &\Leftrightarrow m_1 + n_2 = m_2 + n_1\end{aligned}$$

Here $[S] / \sim$ is the *quotient* of S with respect of equivalence relation \sim , i.e., the decomposition of S into equivalence classes such that two elements are in the same class if and only if they are related by \sim . One such class (the one representing the number 3) is thus $\{(3, 0), (4, 1), (5, 2), (6, 3), \dots\}$.

Every occurrence of a pair (m, n) in an axiom of Solution 2 then has to be interpreted as an occurrence $[(m, n)]$, the equivalence class containing (m, n) . For instance, the interpretation of axiom $(m_1, n_1) + (m_2, n_2) = (m_1 + m_2, n_1 + n_2)$ is

$$[(m_1, n_1)] + [(m_2, n_2)] = [(m_1 + m_2, n_1 + n_2)]$$

Every class has a unique representation which is either of form $[(0, 0)]$ or $[(m, 0)]$ (with $m > 0$) or $[(0, n)]$ (with $n > 0$). Based on this representation, the isomorphism can be defined and the proof of strict adequacy can be performed.