Problems Solved:

31	32	33	34	35

Name:

Matrikel-Nr.:

Problem 31. Let Σ be an alphabet and A be a set ($A \subseteq \Sigma^*$). Let also A be semi-decidable, but not decidable. Prove that the complement of A is not decidable.

Problem 32. Let $M_0, M_1, M_2, \ldots$ be a list of all Turing machines with alphabet $\Sigma = \{0, 1\}$. such that the function $i \mapsto \langle M_i \rangle$ is computable. Let $w_i = 01^i0$ for all natural numbers i. Let $L = \{w_i \mid i \in \mathbb{N} \text{ and } M_i \text{ accepts } w_i\}$ and $\overline{L} = \Sigma^* \setminus L$.

(a) Is L recursively enumerable?

(b) Is $\overline{L}$ recursively enumerable?

(c) Is L recursive?

(d) Is $\overline{L}$ recursive?

Justify your answers.

Problem 33. Let L be a finite language over an alphabet $\{0, 1\}$. Is the following problem

For a Turing maschine M it holds $L(M) \supseteq L$.

semi-decidable? Is it also decidable?

Problem 34. Which of the following problems are decidable? In each problem below, the input of the problem is the code $\langle M \rangle$ of a Turing machine M with input alphabet $\{0, 1\}$.

1. Is $L(M)$ empty?
2. Is $L(M)$ finite?
3. Is $L(M)$ regular?
4. Is $L(M) \subseteq \{0, 1\}^*$?
5. Is $L(M)$ not recursively enumerable?
6. Does M have an even number of states?

Problem 35. (a) Given any Turing machine M, construct a grammar G with the following property:

$$L(G) \neq \emptyset \iff M \text{ halts on the empty input } \epsilon. \quad (1)$$

The construction is supposed to be computable.

Hint: Encode reachable configurations

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                                        +-----+
                                        |     |
                                        |  q  |
                                        |     |
                                        +-----+
                                           |
                                           |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|     |     |     |     |     |     |     |     |     |     |     |
| x_1 | x_2 | ... | x_m | y_1 | y_2 | ... | y_n |  _  |  _  | ...
|     |     |     |     |     |     |     |     |     |     |     |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
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of the Turing machine as the sententials forms

$$\#x_1x_2 \dots x_m q y_1 y_2 \dots y_n \#$$

of G. Simulate transitions of the Turing machine by productions of the grammar.

(b) Is it decidable if a grammar G satisfies $L(G) \neq \emptyset$? (An instance of this decision problem is a grammar coded as a bit string.) Justify your answer.

(c) Is it decidable if two grammars G_1 and G_2 describe the same language? (An instance of this decision problem is a bit string that encodes a pair (G_1, G_2) of grammars.) Justify your answer.