Problems Solved:

26	27	28	29	30

Name:

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Problem 26. Consider the following term rewriting system:

$$p(x, s(y)) \to p(s(x), y) \tag{1}$$
$$p(x, 0) \to x \tag{2}$$

1. Show that
$$p(s(0), s(0)) \xrightarrow{*} s(s(0))$$
by a suitable reduction sequence. For each reduction step, underline the subterm that you reduce, and indicate the reduction rule and the matching substitution σ used explicitly.

2. Disprove that
$$p(p(s(0), s(0)), p(s(0), s(0))) \xrightarrow{*} s(s(0)).$$

Problem 27. Configurations of Turing machines can be encoded as a terms in various ways; for instance we can encode the configuration

```
                                        +-----+
                                        |     |
                                        |  q  |
                                        |     |
                                        +-----+
                                           |
                                           |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+----
|     |     |     |     |     |     |     |     |     |
| x_m | ... | x_2 | x_1 |  z  | y_1 | y_2 | ... | y_n | ...
|     |     |     |     |     |     |     |     |     |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+----
```

as the term

$$g(q, z, f(x_1, f(x_2 \cdots f(x_m, e))), f(y_1, f(y_2 \cdots f(y_m, e)))).$$

In the picture, q is the state of the head and the symbols $x_m, \ldots, x_1; z; y_1, \ldots, y_n \in \Gamma$ describes the tape to the left / under / to the right of the head.
Show how to translate the transition function $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ to a set of term rewrite rules.

1. Give a rewrite rule for each $q \in Q$ and each $c \in \Gamma$ with $\delta(q, c) = (q', c', L)$

2. Give a rewrite rule for each $q \in Q$ and each $c \in \Gamma$ with $\delta(q, c) = (q', c', R)$

Hint: It helps to draw pictures of the machine configuration before and after a transition and to translate both configurations to terms.

Problem 28. Define the following languages by context free grammars over the alphabet $\Sigma = \{0, 1\}$.

(a) $L_1 = \{w \mid w \text{ contains at least two zeroes.}\}$

(b) $L_2 = \{w \mid w \text{ starts and ends with one and the same symbol.}\}$

(c) $L_3 = \{w \mid w \text{ consists of an odd number of symbols and the symbol in the center of } w \text{ is a } 0.\}$

(d) $L_4 = L_2 \cap L_3$

Problem 29. Consider the grammar $G = (N, \Sigma, P, S)$ where $N = \{S\}$, $\Sigma = \{a, b\}$, $P = \{S \to \epsilon, S \to aSbS\}$.

(a) Is $aababb \in L(G)$?

(b) Is $aabab \in L(G)$?

(c) Does every element of $L(G)$ contain the same number of occurrences of a and b?

(d) Is $L(G)$ regular?

(e) Is $L(G)$ recursive?

Justify your answers.

Problem 30. Construct a FSM recognizing $L(G)$ where G is the grammar:

$$S \to aS|bA|b$$

$$A \to aA|bS|a$$