Formal Specification of Abstract Datatypes
Exercise 3 (May 28)

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The result for each exercise is to be submitted by the deadline stated above via the Moodle interface as a .zip or .tgz file which contains

- a PDF file with
  - a cover page with the title of the course, your name, Matrikelnummer, and email-address,
  - the content required by the exercise (specification, source, proof),
- (if required) the CafeOBJ (.mod) file(s) of the specifications.

Exercise 3: Strict Adequacy of Specification

Give a *strictly* adequate specification of the integers (see Exercise 2) and prove that it is indeed strictly adequate w.r.t the "classical" integer algebra with carrier $\mathbb{Z}$. You only need to show the homomorphism condition for the operations 0_i, 1_i, $+_i$, and $=_i$.

Hint: As shown in class, it is easiest to perform the proof, if you have a unique term representation for every element of $\mathbb{Z}$. You may therefore base this exercise on Solution 1 of Exercise 3 which already provides a unique term representation.

Bonus (30%): alternatively you may base this exercise also on Solution 2 by replacing the interpretation $\mathcal{M}(sp)(int) = \mathbb{N} \times \mathbb{N}$ with the interpretation

$$\mathcal{M}(sp)(int) = [\mathbb{N} \times \mathbb{N}]/\sim$$
$$(m_1, n_1) \sim (m_2, n_2) \Leftrightarrow m_1 + n_2 = m_2 + n_1$$

Here $[S]/\sim$ is the *quotient* of S with respect of equivalence relation $\sim$, i.e., the decomposition of S into equivalence classes such that two elements are in the same class if and only if they are related by $\sim$. One such class (the one representing the number 3) is thus $\{(3, 0), (4, 1), (5, 2), (6, 3), \ldots\}$.

Every occurrence of a pair (m, n) in an axiom of Solution 2 then has to be interpreted as an occurrence $[(m, n)]$, the equivalence class containing (m, n). For instance, the interpretation of axiom $(m_1, n_1) + (m_2, n_2) = (m_1 + m_2, n_1 + n_2)$ is

$$[(m_1, n_1)] + [(m_2, n_2)] = [(m_1 + m_2, n_1, n_2)]$$

Every class has a unique term representation which is either of form $[(0, 0)]$ or $[(m, 0)]$ (with $m > 0$) or $[(0, n)]$ (with $n > 0$). Based on this term representation, the isomorphism can be defined and the proof of strict adequacy can be performed.