

**Problems Solved:**

31	32	33	34	35
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**Name:****Matrikel-Nr.:**

**Problem 31.** Define the following languages by context free grammars over the alphabet  $\Sigma = \{0, 1\}$ .

- (a)  $L_1 = \{w \mid w \text{ contains at least two zeroes.}\}$
- (b)  $L_2 = \{w \mid w \text{ starts and ends with one and the same symbol.}\}$
- (c)  $L_3 = \{w \mid w \text{ consists of an odd number of symbols and the symbol in the center of } w \text{ is a } 0.\}$
- (d)  $L_4 = L_2 \cap L_3$

**Problem 32.** Consider the grammar  $G = (N, \Sigma, P, S)$  where  $N = \{S\}$ ,  $\Sigma = \{a, b\}$ ,  $P = \{S \rightarrow \epsilon, S \rightarrow aSbS\}$ .

- (a) Is  $aababb \in L(G)$ ?
- (b) Is  $aabab \in L(G)$ ?
- (c) Does every element of  $L(G)$  contain the same number of occurrences of  $a$  and  $b$ ?
- (d) Is  $L(G)$  regular?
- (e) Is  $L(G)$  recursive?

Justify your answers.

**Problem 33.** Let  $M_0, M_1, M_2, \dots$  be a list of all Turing machines with alphabet  $\Sigma = \{0, 1\}$ . Let  $w_i = 01^i0$  for all natural numbers  $i$ . Let  $L = \{w_i \mid i \in \mathbb{N} \text{ and } M_i \text{ accepts } w_i\}$  and  $\bar{L} = \Sigma^* \setminus L$ .

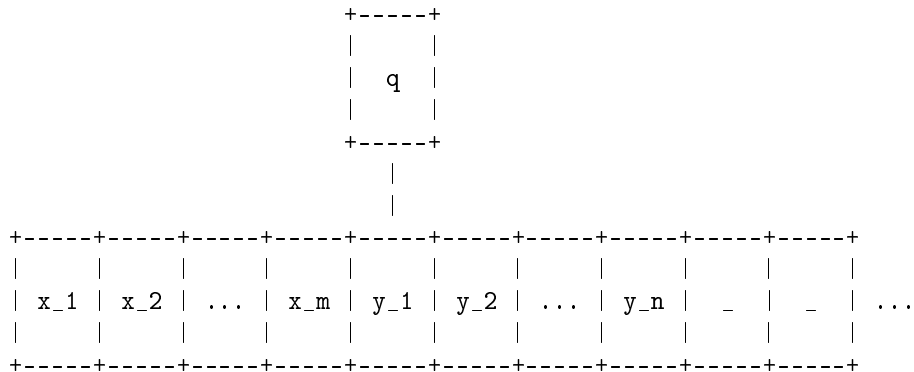
- (a) Is  $L$  recursively enumerable?
- (b) Is  $\bar{L}$  recursively enumerable?
- (c) Is  $L$  recursive?
- (d) Is  $\bar{L}$  recursive?

Justify your answers.

**Problem 34.** (a) Given a Turing machine  $M$ , construct a grammar  $G$  with the following property:

$$L(G) \neq \emptyset \iff M \text{ halts on the empty input } \epsilon. \quad (1)$$

*Hint:* Encode reachable configurations



of the Turing machine as the sentential forms

$$\#x_1x_2 \dots x_mqy_1y_2 \dots y_n\#$$

of  $G$ . Simulate transitions of the Turing machine by productions of the grammar.

- (b) Is it decidable if a grammar  $G$  satisfies  $L(G) \neq \emptyset$ ? (An instance of this decision problem is a grammar coded as a bit string.) Justify your answer.
- (c) Is it decidable if two grammars  $G_1$  and  $G_2$  describe the same language? (An instance of this decision problem is a bit string that encodes a pair  $(G_1, G_2)$  of grammars.) Justify your answer.

**Problem 35.** Which of the following problems are decidable? In each problem below, the input of the problem is the code  $\langle M \rangle$  of a Turing machine  $M$  with input alphabet  $\{0, 1\}$ .

1. Is  $L(M)$  empty?
2. Is  $L(M)$  finite?
3. Is  $L(M)$  regular?
4. Is  $L(M) \subseteq \{0, 1\}^*$ ?
5. Is  $L(M)$  not recursively enumerable?
6. Does  $M$  have an even number of states?