31	32	33	34	35

Problems Solved:

Name:

Matrikel-Nr.:

Problem 31. Define the following languages by context free grammars over the alphabet $\Sigma = \{0, 1\}$.

(a) $L_1 = \{w \mid w \text{ contains at least two zeroes.}\}$

(b) $L_2 = \{w \mid w \text{ starts and ends with one and the same symbol.}\}$

(c) $L_3 = \{w \mid w \text{ consists of an odd number of symbols and the symbol in the ceter of } w \text{ is a } 0.\}$

(d) $L_4 = L_2 \cap L_3$

Problem 32. Consider the grammar $G = (N, \Sigma, P, S)$ where $N = \{S\}$, $\Sigma = \{a, b\}$, $P = \{S \to \epsilon, S \to aSbS\}$.

(a) Is $aababb \in L(G)$?

(b) Is $aabab \in L(G)$?

(c) Does every element of $L(G)$ contain the same number of occurrences of a and b?

(d) Is $L(G)$ regular?

(e) Is $L(G)$ recursive?

Justify your answers.

Problem 33. Let $M_0, M_1, M_2, \ldots$ be a list of all Turing machines with alphabet $\Sigma = \{0, 1\}$. Let $w_i = 01^i0$ for all natural numbers i. Let $L = \{w_i \mid i \in \mathbb{N} \text{ and } M_i \text{ accepts } w_i\}$ and $\overline{L} = \Sigma^* \setminus L$.

(a) Is L recursively enumerable?

(b) Is $\overline{L}$ recursively enumerable?

(c) Is L recursive?

(d) Is $\overline{L}$ recursive?

Justify your answers.

Problem 34. (a) Given a Turing machine M, construct a grammar G with the following property:

$$L(G) \neq \emptyset \iff M \text{ halts on the empty input } \epsilon. \tag{1}$$

Hint: Encode reachable configurations

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                        +-----+
                        |     |
                        |  q  |
                        |     |
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                           |
                           |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|     |     |     |     |     |     |     |     |     |     |
| x_1 | x_2 | ... | x_m | y_1 | y_2 | ... | y_n |  _  |  _  | ...
|     |     |     |     |     |     |     |     |     |     |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
```

of the Turing machine as the sententials forms

$$\#x_1x_2\dots x_mqy_1y_2\dots y_n\#$$

of G. Simulate transitions of the Turing machine by productions of the grammar.

(b) Is it decidable if a grammar G satisfies $L(G) \neq \emptyset$? (An instance of this decision problem is a grammar coded as a bit string.) Justify your answer.

(c) Is it decidable if two grammars G_1 and G_2 describe the same language? (An instance of this decision problem is a bit string that encodes a pair (G_1, G_2) of grammars.) Justify your answer.

Problem 35. Which of the following problems are decidable? In each problem below, the input of the problem is the code $\langle M \rangle$ of a Turing machine M with input alphabet $\{0, 1\}$.

1. Is $L(M)$ empty?
2. Is $L(M)$ finite?
3. Is $L(M)$ regular?
4. Is $L(M) \subseteq \{0, 1\}^*$?
5. Is $L(M)$ not recursively enumerable?
6. Does M have an even number of states?