Problems Solved:

26	27	28	29	30

Name:

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Problem 26. Let $f : \mathbb{N} \to \mathbb{N}$ be defined by $f(n) := 2n + 1$.

1. Show that f is loop computable by giving a loop program that computes f.
2. Show that f is primitive recursive by giving a primitive recursive definition of f.

Problem 27. Let $S : \mathbb{N} \to \{0, 1\}$ be defined by

$$S(x) = \begin{cases} 1 & \text{if } x \text{ is the square of some natural number,} \\ 0 & \text{otherwise.} \end{cases}$$

1. Write down a formal definition of S.
2. Show that S is primitive recursive by giving a primitive recursive definition of S.
3. Show that S is loop computable by giving a loop program that computes S.

Hint: It is OK to assume that the equality check and multiplication are primitive recursive as well as loop computable. The equality check e is given by $e(x, y) = 1$ if $x = y$ and $e(x, y) = 0$ otherwise Any predicate P can be encoded as a function f. We define $f(x) := 1$ if $P(x)$ is true and $f(x) := 0$ if $P(x)$ is false.

Problem 28. Let $f : \mathbb{N} \to_p \mathbb{N}$ be the partial function given by

$$f(x) = y \text{ such that } x = y^2 \text{ if such a } y \text{ exists,}$$
$$f(x) \text{ is undefined, if no such } y \text{ exists.}$$

1. Show that f is while computable. *Hint:* You may use multiplication and equality checks in your while program, because they are known to be while computable.
2. Is f a recursive functions? (Justify your answer. If the answer is *yes*, give an explicit recursive definition of f.)
3. Is f loop computable? (Justify your answer.)
4. Is f a primitive recursive function?(Justify your answer.)

Problem 29. Consider the following term rewriting system:

$$p(x, s(y)) \to p(s(x), y) \tag{1}$$
$$p(x, 0) \to x \tag{2}$$

1. Show that
$$p(s(0), s(0)) \xrightarrow{*} s(s(0))$$
by a suitable reduction sequence. For each reduction step, underline the subterm that you reduce, and indicate the reduction rule and the matching substitution σ used explicitly.

2. Disprove that
$$p(p(s(0), s(0)), p(s(0), s(0))) \xrightarrow{*} s(s(0)).$$

Problem 30. Configurations of Turing machines can be encoded as a terms in various ways; for instance we can encode the configuration

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                        +-----+
                        |     |
                        |  q  |
                        |     |
                        +-----+
                           |
                           |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+----
|     |     |     |     |     |     |     |     |     |
| x_m | ... | x_2 | x_1 |  z  | y_1 | y_2 | ... | y_n | ...
|     |     |     |     |     |     |     |     |     |
+-----+-----+-----+-----+-----+-----+-----+-----+-----+----
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as the term
$$g(q, z, f(x_1, f(x_2 \cdots f(x_m, e))), f(y_1, f(y_2 \cdots f(y_m, e)))).$$

In the picture, q is the state of the head and the symbols $x_m, \ldots, x_1; z; y_1, \ldots, y_n \in \Gamma$ describes the tape to the left / under / to the right of the head.
Show how to translate the transition function $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ to a set of term rewrite rules.

1. Give a rewrite rule for each $q \in Q$ and each $c \in \Gamma$ with $\delta(q, c) = (q', c', L)$
2. Give a rewrite rule for each $q \in Q$ and each $c \in \Gamma$ with $\delta(q, c) = (q', c', R)$

Hint: It helps to draw pictures of the machine configuration before and after a transition and to translate both configurations to terms.