Comparison between Boogie2 and Why3 for the verification of *MiniMaple* programs

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Outline

1. Introduction
2. Boogie2
3. Why3
4. My Work
5. Conclusions

Introduction

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- *n* programming languages and
- *m* theorem provers

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For program verification, we need

- $n \times m$ translations to generate verification conditions

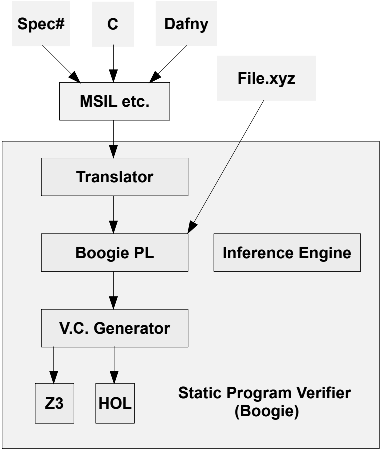
Better solution is to translate n programs

- into a common intermediate (verification) language
    - common to m provers
- requires $n + m$ translations
- benefits
    - automatically generate verification conditions
    - these conditions can be proved by the prover of one's choice

Intermediate Verification Languages

- Boogie (by Microsoft 2006)
    - SPEC#
    - VCC
    - Dafny
- Why (by LRI, France 2003)
    - Krakatoa
    - Caduceus
    - Frama-C

Boogie2



Spec#

C

Dafny

File.xyz

MSIL etc.

Translator

Boogie PL

Inference Engine

V.C. Generator

Z3

HOL

Static Program Verifier
(Boogie)

Elements of the Boogie Language

- Mathematical components specify
  - types
  - constants
  - functions
  - axioms
- Imperative components specify
  - global variables
  - procedure declarations
  - procedure implementations
  - also described respective constrained states by mathematical components
  - sets of execution traces
    - e.g. in case of procedure, caller and callee traces
- Expressions
  - typical logical, boolean, arithmetic etc.

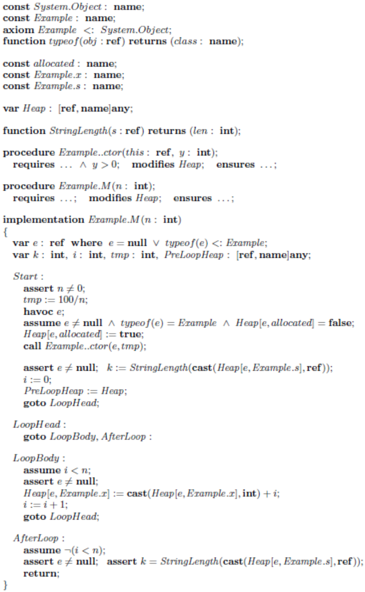
Features of the Boogie Language

- Parametric polymorphism
  - function can take polymorphic type parameters
- Partial ordering ($<:$)
  - for constants of the same type
- Nondeterminism (havoc statements)
  - can assign arbitrary values to a set of variables
- Flow-chart like language
  - non-imperative and only supports goto statements

An Example Spec# Programs

```
public class Example {
  int x;
  string! s;
  invariant s.Length >= 12;
  public Example(int y) requires y > 0; { ... }
  public static void M(int n) {
    Example e = new Example(100/n);
    int k = e.s.Length;
    for (int i = 0; i < n; i++) { e.x += i; }
    assert k == e.s.Length;
  }
}
```

An Example Spec# to Boogie Translation



```
const System.Object : name;
const Example : name;
axiom Example <: System.Object;
function typeof(obj : ref) returns (class : name);

const allocated : name;
const Example.x : name;
const Example.s : name;

var Heap : [ref, name]any;

function StringLength(s : ref) returns (len : int);

procedure Example..ctor(this : ref, y : int);
  requires ... ∧ y > 0;   modifies Heap;   ensures ...;

procedure Example.M(n : int);
  requires ...;   modifies Heap;   ensures ...;

implementation Example.M(n : int)
{
  var e : ref where e = null ∨ typeof(e) <: Example;
  var k : int, i : int, tmp : int, PreLoopHeap : [ref, name]any;

  Start :
    assert n ≠ 0;
    tmp := 100/n;
    havoc e;
    assume e ≠ null ∧ typeof(e) = Example ∧ Heap[e, allocated] = false;
    Heap[e, allocated] := true;
    call Example..ctor(e, tmp);

    assert e ≠ null;   k := StringLength(cast(Heap[e, Example.s], ref));
    i := 0;
    PreLoopHeap := Heap;
    goto LoopHead;

  LoopHead :
    goto LoopBody, AfterLoop :

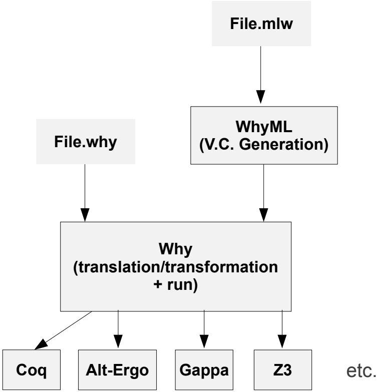
  LoopBody :
    assume i < n;
    assert e ≠ null;
    Heap[e, Example.x] := cast(Heap[e, Example.x], int) + i;
    i := i + 1;
    goto LoopHead;

  AfterLoop :
    assume ¬(i < n);
    assert e ≠ null;   assert k = StringLength(cast(Heap[e, Example.s], ref));
    return;
}
```

Strengths and Weaknesses

- Strengths
  - many front-end tools support Boogie
    - Spec# compiler
  - imperative style syntax
- Weaknesses
  - no rich theory language
  - full verification is hard
    - only have very good supports of Z3
  - no sufficiently documented semantics definition

Why3


File.mlw
WhyML
(V.C. Generation)
File.why
Why
(translation/transformation
+ run)
Coq
Alt-Ergo
Gappa
Z3
etc.

Simplicity and Collaborative Proofs

- Generates simple verification conditions
  - no memory store
  - conditions about the contents of the data structures
- Still captures sufficient details
  - termination and array bound checking etc.
- Provides collaborative proofs
  - to handle unproved verification conditions with interactive provers
  - but provides as much proof automation as possible
- Also WP-based semantics

Some more features

Influenced by ML

- Why3 supports
  - algebraic data types
  - pattern matching
- WhyML supports
  - type inference
  - currying
  - abstract data types

Theories and Modules

- Built-in theories
  - e.g., List, Int etc.
- Built-in modules
  - e.g., Ref etc.
- Can be used directly or by cloning

```
(* Theory Definition *)
theory Orty
        use import list.List

        type orty
        ...
end

(* Module Definition *)
module MyModule

        use import int.Int
        use import module ref.Ref
        use import Orty

        ...
end
```

Abstract and Algebraic Data Types

```
(* Abstract Data Type *)
theory Orty

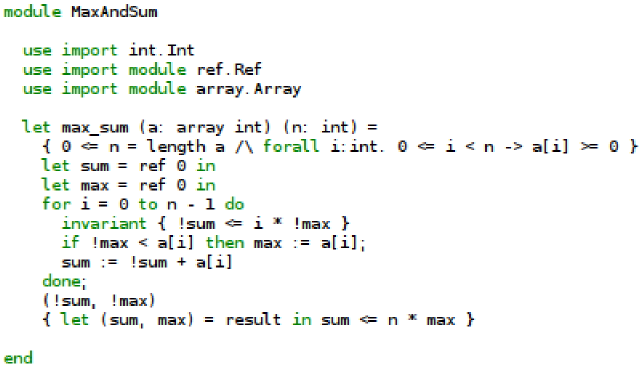
        type orty
        ...
end

(* Algebraic Data Type *)
theory List

        type list 'a = Nil | Cons 'a (list 'a)

end
```

An Example Why3 Program - MaxAndSum



```
module MaxAndSum

  use import int.Int
  use import module ref.Ref
  use import module array.Array

  let max_sum (a: array int) (n: int) =
    { 0 <= n = length a /\ forall i:int. 0 <= i < n -> a[i] >= 0 }
    let sum = ref 0 in
    let max = ref 0 in
    for i = 0 to n - 1 do
      invariant { !sum <= i * !max }
      if !max < a[i] then max := a[i];
      sum := !sum + a[i]
    done;
    (!sum, !max)
    { let (sum, max) = result in sum <= n * max }

end
```

Strengths and Weaknesses

- Strengths
    - rich logic, readily usable in programs
    - support collaborative proofs by many beck-end provers
    - modularity and abstract data types
    - close to specification-based programming
- Weaknesses
    - program and specification are tied together
        - even w.r.t. syntax
    - some data structures cannot be defined (but signatures)
        - e.g. mutable trees etc.

My Work

- Formal specification respectively verification of programs written in (the most widely used) untyped computer algebra languages
  - Mathematica and Maple
- Develop a tool to find errors by static analysis
  - for example type inconsistencies
  - and violations of methods preconditions
- Also
  - to realize the gap between the example computer algebra algorithm and its implementation
  - to formalize properties of computer algebra
- Demonstration example
  - Maple package *DifferenceDifferential* developed by Christian Dönch
- *MiniMaple*
  - A simple but substantial subset of Maple
  - Covers all syntactic domains of Maple but fewer expressions

A *MiniMaple* Example Program

```
sumproc := proc(l: Or(integer, list(integer)))::integer;
           local sum::integer:=0, el::list(integer), x::integer;
           if type(l,integer) then
                 if l <> 0 then
                           sum := sum + l;
                 else
                      return sum;
                 end if;
           elif type(l,list(integer)) then
                   for x from 1 by 1 to nops(l) do
                        el := l[x];
                        if el <> 0 then
                             sum:=sum+el;
                        else
                             return sum;
                        end if;
                   end do;
           end if;
           return sum;
end proc;
```

Special features of the *MiniMaple* Type System

- Uses only *Maple* type annotations
  - *Maple* uses them for *dynamic type checking*
  - *MiniMaple* uses them for *static type checking*
- Context (global vs local)
  - *global*
    - may introduce new identifiers by assignments
    - types of identifiers may change arbitrarily by assignments
  - *local*
    - identifiers only introduced by declarations
    - types of identifiers can only be *specialized*
- Type tests in Maple, i.e. **type**(I, T)
  - branches may have different type information for the same variable
    - track type information to allow satisfiable tests only
  - number of loop iterations might influence the type information
    - least fix point as an upper bound on the types of the variable
    - as a special case the declared type is the least fixed point

Elements of the Specification Language

- Mathematical theories
  - Types
    - User defined data-types
    - Abstract data types
  - Functions and predicates (declared/defined)
  - Axioms
- Procedure specifications
  - Pre-post conditions
  - Exceptions
  - Global variables
- Loop specifications
  - Invariants
  - Termination terms
- Assertions
  - To constrain the state of execution

Challenges of Specification Language for *MiniMaple*

- Support of some non-standard types of objects
  - e.g. symbols, unevaluated expressions etc.
- Additional functions and predicates
  - e.g. type test, **type**(I, T)
- Specification of abstract mathematical concepts by an abstract data type
  - Weaker support in current classical specification languages
  - e.g., ring, variables and ordering of a polynomial
  - ADDO as an abstract data type represented by list of tuples
    - Abstract Difference Differential Operator

An example utility procedure of *DifferenceDifferential*

```
(*@
    'type/ADDO';
    define(terms, terms(ad::ADDO)=...);
    define(getTerm, getTerm(ad::ADDO,i::nat, j::nat)=...);
    isADDO(d);
    isADDOTerm(c,n,z,e);
    ...
    assume(isADDO(d) equivalent forall(i::integer, 1<=i and i<=terms(d) implies
             isADDOTerm(getTerm(d,i,1), getTerm(d,i,2), getTerm(d,i,3), getTerm(d,i,4]));
    assume(isADDOTerm(c,n,z,e) equivalent inField(c) and isGenerator(e));
   ...
    define(power, power(a::integer,0)=1, power(a::integer,b::integer)= mul(a,1...b));
    define(maps, maps(d::DDO)=...);
   @*)
global noauto, generators, ...;
   ...
(*@
    requires 1 <= z and z <= power(2,length(noauto)) and
              forall(i::integer, 1<=i and i<=terms(maps(a)) implies isGenerator(getTerm(maps(a),i,4))) and
              forall(i::integer, 1<=i and i<=terms(maps(b)) implies isGenerator(getTerm(maps(b),i,4)));
    global EMPTY;
    ensures
       ( forall(j::integer, 1<=j and j<=nops(RESULT) implies isGenerator(RESULT[j][1],maps(a),maps(b)) and
                      RESULT[j][2] = isLT(maps(a),z) and RESULT[j][3] = isLT(maps(b),z)) )
       or
       (nops(RESULT) = 0 and ...);
    @*)

VGB := proc (z::integer, a::DDO, b::DDO)::list([symbol,list(symbol),list(symbol)]) ... return v; end proc;
```

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Need to verify the implementation of some computer algebra algorithm along-with reasonable proof/details about the algorithm itself

- *MiniMaple* and its specification language
  - symbolic programs are close to algorithms
- Arguments in favor of Why3
  - rich theory language
  - algebraic and abstract data types
  - inductive predicates
  - both automated and interactive proof

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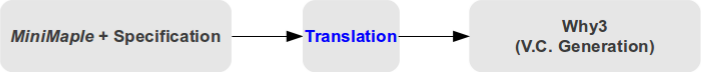
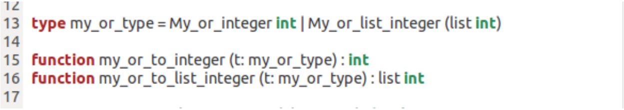


MiniMaple + Specification → Translation → Why3 (V.C. Generation)

Translation to corresponding semantically equivalent Why3 constructs

An Example Translation (*MiniMaple* to Why3)

- Union-type, i.e. Or(integer, list(integer))



```
type my_or_type = My_or_integer int | My_or_list_integer (list int)

function my_or_to_integer (t: my_or_type) : int
function my_or_to_list_integer (t: my_or_type) : list int

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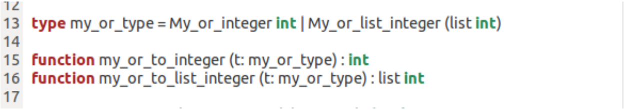

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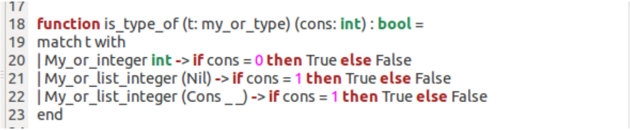

```
function is_type_of (t: my_or_type) (cons: int) : bool =
match t with
| My_or_integer int -> if cons = 0 then True else False
| My_or_list_integer (Nil) -> if cons = 1 then True else False
| My_or_list_integer (Cons _ _) -> if cons = 1 then True else False
end
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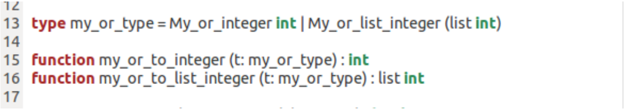

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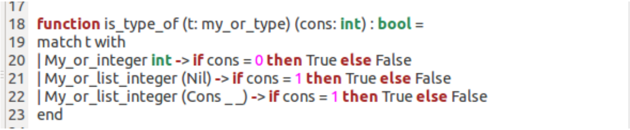

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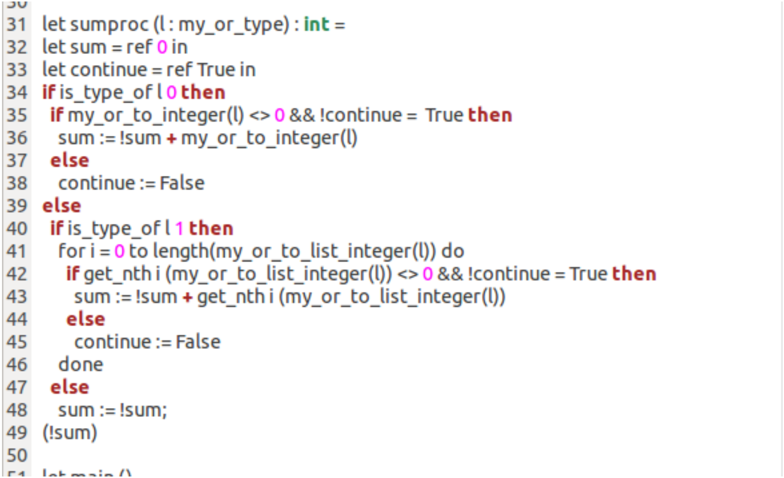
end
```

- Utility function to extract *nth* element of a list

```
let get_nth (i: int) (l: list int) =
 match nth i l with
 | None -> absurd
 | Some x -> x
 end
```

MiniMaple to Why3 - contd.

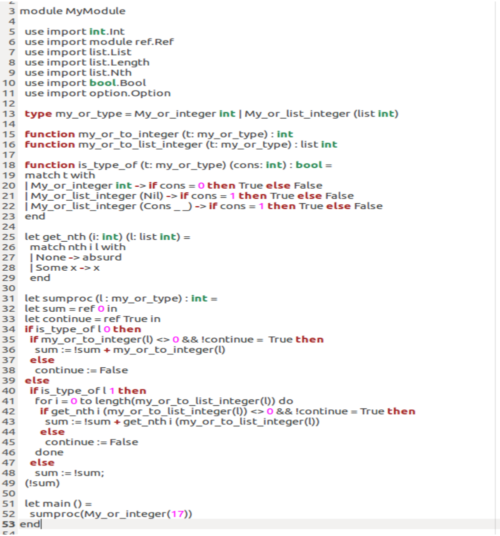
- Procedure sumproc(l: Or(integer, list(integer)))::integer



```
let sumproc (l : my_or_type) : int =
let sum = ref 0 in
let continue = ref True in
if is_type_of l 0 then
 if my_or_to_integer(l) <> 0 && !continue =  True then
  sum := !sum + my_or_to_integer(l)
 else
  continue := False
else
 if is_type_of l 1 then
  for i = 0 to length(my_or_to_list_integer(l)) do
   if get_nth i (my_or_to_list_integer(l)) <> 0 && !continue = True then
    sum := !sum + get_nth i (my_or_to_list_integer(l))
   else
    continue := False
  done
 else
  sum := !sum;
(!sum)

let main ()
```

Complete Example Translation



```
module MyModule

 use import int.Int
 use import module ref.Ref
 use import list.List
 use import list.Length
 use import list.Nth
 use import bool.Bool
 use import option.Option

 type my_or_type = My_or_integer int | My_or_list_integer (list int)

 function my_or_to_integer (t: my_or_type) : int
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 else
  if is_type_of l 1 then
   for i = 0 to length(my_or_to_list_integer(l)) do
    if get_nth i (my_or_to_list_integer(l)) <> 0 && !continue = True then
     sum := !sum + get_nth i (my_or_to_list_integer(l))
    else
     continue := False
   done
  else
   sum := !sum;
 (!sum)

 let main () =
  sumproc(My_or_integer(17))
end
```

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    - type-tests
    - sub-typing relations
  - Other constructs (supported by the corresponding constructs)
    - procedures, modules
    - for-loop variations
    - exception handling

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  - Loop specifications (supported by invariants + variants)
    - invariants
    - termination term

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  - Assertions (supported)

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  - Other constructs (supported)
    - typed logical quantifiers