Introduction to Parallel and Distributed Computing Exercise 4 (June 29, 2026)

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The result is to be submitted by the deadline stated above via the Moodle interface as a .zip or .tgz file which contains

- a single PDF (`.pdf`) file with
  - a cover page with the title of the course, your name, matriculation number, and email address,
  - a section with the source code of the program benchmarked, the output of the parallelizing compiler, and an explanation of the output,
  - a section with the raw data of the benchmarks,
  - a section with a summary table and graphical diagrams of the benchmarks.
- the source (`.c`/`.cpp`) file(s) of the programs.

Exercise 4: Message Passing Programming in MPI

The goal of this exercise is to develop a MPI-based parallel solution to the "all pairs shortest paths" problem described in Exercise 1.

MPI Program Initially process 0 broadcasts $D := W$ to every process (`MPI_Bcast`). The program then runs in multiple rounds where in each round $D := D \times D$ is computed as follows:

- every process computes some rows of $D \times D$ (row-wise parallelization),
- every process gathers the results of all other processes (`MPI_Allgather`) such that every process holds again the complete $D := D \times D$.

The dimension n of the matrix is not necessarily a multiple of the number n of processes. To simplify the gathering, one may thus extend the matrix to a dimension $n' = p \cdot \lceil n/p \rceil$ and let n'/p rows be gathered from every process.

Scalability Analysis Furthermore, perform a scalability analysis of the program, i.e.:

- determine the basic execution time T_n;
- determine the parallelization overhead $P_{p,n}$;
- determine the solution n_p of the constraint $T_{n_p} = K_E \cdot P_{p,n_p}$;
- determine the isoefficiency function $I_p^E = K_E \cdot P_{p,n_p}$.

For determining $P_{p,n}$, you just need to consider the communication overhead. Here it suffices to use a simple communication model where sending a message of size m takes time $\Theta(m)$. Furthermore assume that broadcasting a message to p processors is implemented by sending p individual messages to each processor, i.e., takes time $\Theta(p \cdot m)$; likewise, scattering a message of size m among p processors takes the same time as p times broadcasting a message of size m/p, i.e., it takes time $\Theta(p \cdot p \cdot m/p) = \Theta(p \cdot m)$.

Benchmarking Finally, benchmark the program as follows:

- Take the sequential solution and benchmark it with two appropriate values N_1, N_2 for the matrix dimension (at least one should run for at least one minute).
- Benchmark the MPI version of the program for N_1 and N_2 and $P = 1, 4, 8, 16, 32, 64$ processes. Do not forget to set the environment variable `MPI_DSM_CPULIST` to pin processes to separate physical processor cores.
- Apply the result of the scalability analysis to scale the larger of N_1 or N_2 for $P = 1, 4, 8, 16, 32, 64$ processors; benchmark for these values both the sequential and the parallel program. Do the benchmark results confirm the results of the scalability analysis (i.e., is the efficiency preserved at a high level)?

Perform your benchmarks and present the results in the same way as in Exercise 1.