

We prove:

pass[Jack, 4] (pass Jack)

under the assumptions:

$\forall_{s, T} \text{pass}[s, T] \Leftrightarrow \text{passBE}[s, T] \vee \text{passWQ}[s, T],$ (WQ or BE)

$\forall_{s, T} \text{passWQ}[s, T] \Leftrightarrow \left(\forall_{t=1, \dots, T} \text{passTopic}[s, t] \right) \wedge \text{passTotal}[s, T] \wedge (\text{mode}[s] == \text{WQ}),$ (pass WQ)

$\forall_{s, t} \text{passTopic}[s, t] \Leftrightarrow \text{scoreTopic}[s, t] \geq 7.5,$ (pass WQ topic)

$\forall_{s, t} \text{scoreTopic}[s, t] := \left(\sum_{m=3 \text{ } t-2, \dots, 3 \text{ } t} \text{score}[\text{ModuleP}[s], m] \right),$ (pass WQ topic)

$\forall_s \text{ModuleP}[s] := \left(\min \left\{ \text{QuizP}[s]_i + \text{BonusP}[s]_i, 10 \right\} \mid_{i=1, \dots, |\text{QuizP}[s]|} \right),$ (module points)

$\forall_{s, T} \text{passTotal}[s, T] \Leftrightarrow \left(\sum_{t=1, \dots, T} \text{scoreTopic}[s, t] \right) \geq 60,$ (pass total)

$\forall_{S, m} \text{score}[S, m] := S_m,$ (extract score)

mode[Jack] := WQ, (Jack mode)

QuizP[Jack] := <5, 5, 4, 7, 5, 3, 8, 8, 1, 10, 2, 5>, (Jack quiz)

BonusP[Jack] := <0.5, 0.3, 1, 0.2, 0.1, 1, 0.4, 0.4, 0.7, 0.3, 0.2, 0.6>. (Jack bonus)

We expand definitions:

In order to prove (pass Jack), using definition (WQ or BE), we now show

passBE[Jack, 4] $\vee$ passWQ[Jack, 4]. (G#6)

For proving the disjunction (G#6) we prove

passWQ[Jack, 4]. (G#7)

We expand definitions:

In order to prove (G#7), using definition (pass WQ), we now show

(passTopic[Jack, 1] $\wedge$ passTopic[Jack, 2] $\wedge$ passTopic[Jack, 3] $\wedge$ passTopic[Jack, 4]) $\wedge$ passTotal[Jack, 4] $\wedge$ (mode[Jack] == WQ). (G#13)

For proving (G#13) we prove the individual conjuncts.

☑ Proof of (G#13.1):

We need to prove

passTopic[Jack, 1] $\wedge$ passTopic[Jack, 2] $\wedge$ passTopic[Jack, 3] $\wedge$ passTopic[Jack, 4]. (G#13.1)

For proving (G#13.1) we prove the individual conjuncts.

☑ Proof of (G#13.1.1):

We need to prove

passTopic[Jack, 1]. (G#13.1.1)

We expand definitions:

In order to prove (G#13.1.1), using definition (pass WQ topic), we now show

scoreTopic[Jack, 1] ≥ 7.5. (G#32)

We expand definitions:

In order to prove (G#32), using definition (pass WQ topic), we now show

(score[ModuleP[Jack], 1] + score[ModuleP[Jack], 2] + score[ModuleP[Jack], 3]) ≥ 7.5. (G#38)

We expand definitions:

In order to prove (G#38), using definition (extract score), we now show

(ModuleP[Jack]₁ + ModuleP[Jack]₂ + ModuleP[Jack]₃) ≥ 7.5. (G#39)

We expand definitions:

In order to prove (G#39), using definition (module points), we now show

$$\left(\min_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} [\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10] \right)_1 + \left(\min_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} [\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10] \right)_2 + \left(\min_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} [\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10] \right)_3 \geq 7.5. \quad (\text{G\#40})$$

We apply substitutions:

In order to prove (G#40), using (Jack quiz) and (Jack bonus), we now show

True. (G#41)

This part of the proof is now finished, because the goal (G#41) is true.

☑ Proof of (G#13.1.2):

We need to prove

passTopic[Jack, 2]. (G#13.1.2)

We expand definitions:

In order to prove (G#13.1.2), using definition (pass WQ topic), we now show

scoreTopic[Jack, 2] ≥ 7.5. (G#42)

We expand definitions:

In order to prove (G#42), using definition (pass WQ topic), we now show

(score[ModuleP[Jack], 4] + score[ModuleP[Jack], 5] + score[ModuleP[Jack], 6]) ≥ 7.5. (G#48)

We expand definitions:

In order to prove (G#48), using definition (extract score), we now show

(ModuleP[Jack]₄ + ModuleP[Jack]₅ + ModuleP[Jack]₆) ≥ 7.5. (G#49)

We expand definitions:

In order to prove (G#49), using definition (module points), we now show

$$\left(\min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right)_4 +$$

$$\left(\min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right)_5 +$$

$$\left(\min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right)_6 \geq 7.5. \quad (\text{G\#50})$$

We apply substitutions:

In order to prove (G#50), using (Jack quiz) and (Jack bonus), we now show

True. (G#51)

This part of the proof is now finished, because the goal (G#51) is true.

☑ Proof of (G#13.1.3):

We need to prove

passTopic[Jack, 3]. (G#13.1.3)

We expand definitions:

In order to prove (G#13.1.3), using definition (pass WQ topic), we now show

scoreTopic[Jack, 3] ≥ 7.5. (G#52)

We expand definitions:

In order to prove (G#52), using definition (pass WQ topic), we now show

(score[ModuleP[Jack], 7] + score[ModuleP[Jack], 8] + score[ModuleP[Jack], 9]) ≥ 7.5. (G#58)

We expand definitions:

In order to prove (G#58), using definition (extract score), we now show

(ModuleP[Jack]₇ + ModuleP[Jack]₈ + ModuleP[Jack]₉) ≥ 7.5. (G#59)

We expand definitions:

In order to prove (G#59), using definition (module points), we now show

$$\left(\min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right)_7 +$$

$$\left(\min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right)_8 +$$

$$\left(\min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right)_9 \geq 7.5. \quad (\text{G\#60})$$

We apply substitutions:

In order to prove (G#60), using (Jack quiz) and (Jack bonus), we now show

True. (G#61)

This part of the proof is now finished, because the goal (G#61) is true.

☑ Proof of (G#13.1.4):

We need to prove

$$\text{passTopic}[\text{Jack}, 4]. \quad (\text{G\#13.1.4})$$

We expand definitions:

In order to prove (G#13.1.4), using definition (pass WO topic), we now show

$$\text{scoreTopic}[\text{Jack}, 4] \geq 7.5. \quad (\text{G\#62})$$

We expand definitions:

In order to prove (G#62), using definition (pass WO topic), we now show

$$(\text{score}[\text{ModuleP}[\text{Jack}], 10] + \text{score}[\text{ModuleP}[\text{Jack}], 11] + \text{score}[\text{ModuleP}[\text{Jack}], 12]) \geq 7.5. \quad (\text{G\#68})$$

We expand definitions:

In order to prove (G#68), using definition (extract score), we now show

$$(\text{ModuleP}[\text{Jack}]_{10} + \text{ModuleP}[\text{Jack}]_{11} + \text{ModuleP}[\text{Jack}]_{12}) \geq 7.5. \quad (\text{G\#69})$$

We expand definitions:

In order to prove (G#69), using definition (module points), we now show

$$\begin{aligned} & \left(\min_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} [\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10] \right)_{10} + \\ & \left(\min_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} [\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10] \right)_{11} + \\ & \left(\min_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} [\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10] \right)_{12} \geq 7.5. \end{aligned} \quad (\text{G\#70})$$

We apply substitutions:

In order to prove (G#70), using (Jack quiz) and (Jack bonus), we now show

$$\text{True}. \quad (\text{G\#71})$$

This part of the proof is now finished, because the goal (G#71) is true.

☑ Proof of (G#13.2):

We need to prove

$$\text{passTotal}[\text{Jack}, 4]. \quad (\text{G\#13.2})$$

We expand definitions:

In order to prove (G#13.2), using definition (pass total), we now show

$$(\text{scoreTopic}[\text{Jack}, 1] + \text{scoreTopic}[\text{Jack}, 2] + \text{scoreTopic}[\text{Jack}, 3] + \text{scoreTopic}[\text{Jack}, 4]) \geq 60. \quad (\text{G\#72})$$

We expand definitions:

In order to prove (G#72), using definition (pass WO topic), we now show

$$\begin{aligned}
& (\text{score}[\text{ModuleP}[\text{Jack}], 1] + \text{score}[\text{ModuleP}[\text{Jack}], 2] + \text{score}[\text{ModuleP}[\text{Jack}], 3] + \\
& \quad \text{score}[\text{ModuleP}[\text{Jack}], 4] + \text{score}[\text{ModuleP}[\text{Jack}], 5] + \text{score}[\text{ModuleP}[\text{Jack}], 6] + \\
& \quad \text{score}[\text{ModuleP}[\text{Jack}], 7] + \text{score}[\text{ModuleP}[\text{Jack}], 8] + \text{score}[\text{ModuleP}[\text{Jack}], 9] + \\
& \quad \text{score}[\text{ModuleP}[\text{Jack}], 10] + \text{score}[\text{ModuleP}[\text{Jack}], 11] + \text{score}[\text{ModuleP}[\text{Jack}], 12]) \geq 60.
\end{aligned}
\tag{G\#78}$$

We expand definitions:

In order to prove (G#78), using definition (extract score), we now show

$$\begin{aligned}
& (\text{ModuleP}[\text{Jack}]_1 + \text{ModuleP}[\text{Jack}]_2 + \text{ModuleP}[\text{Jack}]_3 + \text{ModuleP}[\text{Jack}]_4 + \text{ModuleP}[\text{Jack}]_5 + \\
& \quad \text{ModuleP}[\text{Jack}]_6 + \text{ModuleP}[\text{Jack}]_7 + \text{ModuleP}[\text{Jack}]_8 + \text{ModuleP}[\text{Jack}]_9 + \\
& \quad \text{ModuleP}[\text{Jack}]_{10} + \text{ModuleP}[\text{Jack}]_{11} + \text{ModuleP}[\text{Jack}]_{12}) \geq 60.
\end{aligned}
\tag{G\#79}$$

We expand definitions:

In order to prove (G#79), using definition (module points), we now show

$$\begin{aligned}
& \left(\left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_1 + \right. \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_2 + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_3 + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_4 + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_5 + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_6 + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_7 + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_8 + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_9 + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_{10} + \\
& \quad \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_{11} + \\
& \quad \left. \left\langle \min[\{\text{QuizP}[\text{Jack}]_i + \text{BonusP}[\text{Jack}]_i, 10\}] \mid_{i=1, \dots, |\text{QuizP}[\text{Jack}]|} \right\rangle_{12} \right) \geq 60.
\end{aligned}
\tag{G\#80}$$

We apply substitutions:

In order to prove (G#80), using (Jack quiz) and (Jack bonus), we now show

$$\text{True}.
\tag{G\#81}$$

This part of the proof is now finished, because the goal (G#81) is true.

☑ Proof of (G#13.3):

We need to prove

mode[Jack] == WQ.

(G#13.3)

We apply substitutions:

In order to prove (G#13.3), using (Jack mode), we now show

True.

(G#82)

This part of the proof is now finished, because the goal (G#82) is true.