Project Proposal
TaxoLogic

Submitted in the frame of
FFG-Call
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Wolfgang Windsteiger
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The Starting Point

Current Large Language Models (LLM):

- capable of processing and paraphrasing vast amounts of text BUT
- cannot reliably guarantee transparent, auditable, and legally defensible reasoning.

In domains where every decision must be
explained, justified, and withstand regulatory scrutiny,
the black-box character of probabilistic models becomes a critical weakness.

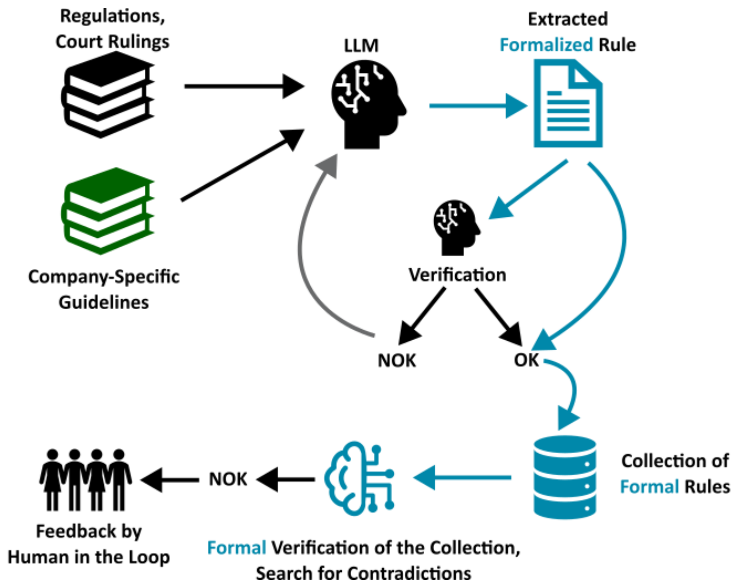
- Example: Tax law (transfer pricing, i.e., valuation of cross-border transactions within multinational enterprises)
  - Case descriptions and regulations in natural language, but
  - their interpretation is ultimately grounded in logic.

Project Idea

- Computer-support for this problem must go beyond the purely statistical learning paradigm.
- Hybrid form of AI that combines the strengths of data-driven models with symbolic reasoning.
- Sub-symbolic components such as LLMs:
  - decompose fiscal requirements into clearly distinct, verifiable individual statements and
  - transform them into machine readable representations.
- Symbolic reasoning operates on these representations to ensure that
  - legal rules, standards, and guidelines are applied consistently,
  - conflicts and exceptions are resolved explicitly, and
  - every inference step can be verified.

In this way, the efficiency and flexibility of LLMs are embedded
within a framework that guarantees logical consistency and delivers reasoning paths that are
transparent, traceable, and defensible before regulators, auditors, and courts.

At One Glance ...



Project Goals (Symbolic Part) I

- Apply rules in a controlled and logically sound fashion.
- As a logical basis we build upon the language of first-order predicate logic (FOL).
- Many reasoning tools available from computer science and mathematics are grounded in FOL.
- Versatile language with strong expressiveness at the cost of undecidability, i.e., there is no method that can, for arbitrary statements in FOL, decide whether the statement is true or false.
- Still, there are methods to formally prove statements provided they are true.
- Automation may become more powerful if we restrict the full power of FOL to certain subsets of the language (Horn-clauses → Prolog, modal or deontic logic?).

Project Goals (Symbolic Part) II

- Automated translation will result in formula-material, which then needs to be consolidated, analyzed, augmented, and corrected. SAT- and SMT-based methods can be employed in order to automatically detect and explain contradictions within the formula set.

Project Goals (Symbolic Part) III

- Contradictions will most probably remain in the formalized laws due to the inherent inconstancy of law → need specially adopted reasoning methods using the obtained formulas, because standard FOL reasoners are based on consistent, i.e., contradiction-free, theories.
- For instance, we cannot apply methods that use reasoning via “proof by contradiction”, a perfectly valid and often-used technique in mathematics.
- This excludes all SAT-based methods from this part of reasoning, because they are all essentially based on proof by contradiction.
- The goal within TaxoLogic is to find and adapt suitable reasoning methods that are resilient to inconsistent rule-bases.

Project Goals (Symbolic Part) IV

- Reasoning methods need to deliver explanations.
- Symbolic methods are trustworthy “by design”, because they are built upon proven correct algorithms.
- Still, they do not by default explain every single step they do.
- For the case of reasoning: No off-the-shelf software.
- Adapt natural deduction-based methods in order to have a realistic chance to get human-understandable logic reasoning chains similar to how a tax expert would argue.

Theorema Formalization

DEFINITION (COURSE MODES) ×

In[∘]:= $\forall_{s,T}$ ×

In[∘]:= `pass[s, T] :⟺ (passBE[s, T] ∨ passWQ[s, T])` (WQ or BE) ×

In[∘]:= `passWQ[s, T] :⟺ ((` $\forall_{t=1,\ldots,T}$ `passTopic[s, t]) ∧ passTotal[s, T] ∧ mode[s] == WQ)` (pass WQ) ×

$$\text{passBE}[s, T] :\Longleftrightarrow \left(\sum_{t=1,\ldots,T} \text{score}[\text{ExamP}[s], t] \geq 40 \land \text{mode}[s] == \text{BE}\right)$$ (pass BE)

Definition (WQ Mode)

In[∘]:= $\forall_{s,t,m,T}$

In[∘]:= $\text{passTopic}[s, t] :\Longleftrightarrow \text{scoreTopic}[s, t] \geq 7.5$ (pass WQ topic)

In[∘]:= $\text{scoreTopic}[s, t] := \sum_{m=3t-2,\ldots,3t} \text{score}[\text{ModuleP}[s], m]$ (pass WQ topic)

In[∘]:= $\text{ModuleP}[s] := \left\langle \min[\{\text{QuizP}[s]_i + \text{BonusP}[s]_i, 10\}] \mid_{i=1,\ldots,|\text{QuizP}[s]|} \right\rangle$ (module points)

In[∘]:= $\text{passTotal}[s, T] :\Longleftrightarrow \sum_{t=1,\ldots,T} \text{scoreTopic}[s, t] \geq 60$ (pass total)

Definition (Score)

In[∘]:= $\forall_{S,m}$

In[∘]:= $\text{score}[S, m] := S_m$ (extract score)

Scores (Jack)

In[∘]:= $\text{mode}[\text{Jack}] := \text{WQ}$ (Jack mode)

In[∘]:= $\text{QuizP}[\text{Jack}] := \langle 5, 5, 4, 7, 5, 3, 8, 8, 1, 10, 2, 5\rangle$ (Jack quiz)

In[∘]:= $\text{BonusP}[\text{Jack}] := \langle 0.5, 0.3, 1, 0.2, 0.1, 1, 0.4, 0.4, 0.7, 0.3, 0.2, 0.6\rangle$ (Jack bonus)

THEOREM (JACK PASS)

In[∘]:=
```
pass[Jack, 4]
```

(pass Jack)

☑ Proof of (pass Jack) #1: *Show proof*

knowledge | **built-in** | **prover** | **Restore settings**

?WQ or BE?, *?pass WQ?*, *?pass BE?*, *?pass WQ topic?*, *?pass WQ topic?*, *?module points?*, *?pass total?*, *?extract score?*, *?Jack mode?*, *?Jack quiz?*, *?Jack bonus?*