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Computational Logic, WS 2025/2026,
Exercise sheet 10,
due date: **25 January 2026, 23:59 via Moodle**

**Problem 43 (20 Points)**
Let $R$ be the term rewriting system: $R := \{a \to b, a \to c, b \to a, b \to d\}$

a) Why is $R$ locally confluent, but not confluent? Justify your answer.

b) Add one rewrite rule that would make $R$ confluent.

c) Remove one rewrite rule from the initial $R$ such that it becomes confluent.

d) Is $R$ terminating? Justify your answer.

**Problem 44 (20 Points)**
Let $R$ be the term rewriting system: $R := \{f(f(x)) \to g(x), g(g(x)) \to f(x)\}$.

a) Is $R$ terminating? Justify your answer.

b) Is $R$ confluent? If not, use the Knuth-Bendix Completion Algorithm to make it confluent.

*Hint*: Please note that the completion shall not destroy termination.

**Problem 45 (20 Points)**
Consider the Bubblesort algorithm which takes a list containing $n$ integers and returns as an output a sorted list. As a postcondition consider the following:

$$\forall i < n(x_i \leq x_{i+1})$$

For the case $n = 3$, provide the SMT-LIB encoding which proves that the final state satisfies the postcondition (give the output of running Z3 with this encoding).

**Problem 46 (20 Points)**
Using Fourier-Motzkin Elimination prove that the given system of inequalities has (respectively does not have) solutions (explicitly give some solution, if there is any):

a) $x + y - 2z \geq 2$, $-x - 3y + z \geq 0$, $y + z \geq 1$

b) $x + y + 2z \geq 1$, $-x + y + z \geq 2$, $x - y + z \geq 1$, $-y - 3z \geq 0$

**Problem 47 (20 Points)**
Using congruence closure prove that the formula $F :\Leftrightarrow f(f(a)) = f(a) \wedge a + f(a) \neq f(f(a)) + a$ is unsatisfiable.