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Computational Logic, WS 2025/2026,
Exercise sheet 9,
due date: **18 January 2026, 23:59 via Moodle**

Problem 39 (30 Points)
Let eq be a binary predicate such that $\forall x(eq(x,x))$ and $\forall x \forall y \forall z\ (eq(x,y) \wedge eq(y,z) \Rightarrow eq(x,z))$. Consider the following formulas:

a) $\exists x\ \exists y\ \forall z(eq(x,z) \vee eq(y,z))$

b) $\forall x\ \exists y\ \exists z(\neg eq(y,z) \wedge \neg eq(x,y) \wedge \neg eq(x,z))$

Using the tableau procedure with free variables and unification prove that the conjunction of the two formulas is unsatisfiable.

Problem 40 (20 Points)
Let x, y, z and t be variables; a and b constants. Consider the following set of clauses:

$$\{\neg P(f(x,y),z) \vee R(z), Q(x) \vee \neg R(a), P(t,a) \vee R(a), \neg Q(b)\}$$

a) Prove by first-order resolution that this set is unsatisfiable.

b) Translate this set of clauses into both a set of Horn clauses (using the concrete syntax of facts and rules) and into a Prolog program, if possible; otherwise, explain why such translations are not possible.

Problem 41 (20 Points)
Consider the following logic program:
$r(g(X)) \leftarrow t(X,Y,f(X))$
$t(a,b,f(a)) \leftarrow$
$p(V,W) \leftarrow r(V)$
with query $\leftarrow p(U,b)$.
What will happen when using SLD resolution?

Problem 42 (30 Points)
Formalize each group of sentences, then prove that the last one is a consequence of the others using resolution and paramodulation.

a)
- Alice's friend has a ginger cat.
- Bob is the only person who has a ginger cat.
- Therefore, Bob is Alice's friend.

b)
- Prof Adam only teaches math students.
- Alice and Bob have the same teacher.
- Prof Adam is Alice's teacher.
- Therefore, Bob is a math student.