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Computational Logic, WS 2025/2026,  
Exercise sheet 8,  
due date: **11 January 2026, 23:59 via Moodle**

**Problem 35 (30 Points)**

Consider the following formula:  $(\exists x p(x) \wedge \neg q(x)) \wedge (\forall y \neg p(y) \vee q(y))$ .

First construct its negation and then using the basic tableaux method prove the validity of the negated formula. Compare your results with those generated by the Proof Tree Generator applying the same method.

In case the two proofs are similar, construct one that is substantially different from the PTG result.

Provide also a manual proof for the validity of the negated formula using tableaux with free variables.

**Problem 36 (20 Points)**

Consider the following formulas:

- a)  $A = \forall x \forall y \forall z (p(x, y) \wedge p(y, z) \Rightarrow p(x, z))$
- b)  $B = \forall x \forall y (p(x, y) \Rightarrow p(y, x))$
- c)  $C = \forall x \forall y \forall z (p(x, y) \wedge p(z, y) \Rightarrow p(x, z))$

Using tableaux with free variables prove that  $A$  and  $B$  entail  $C$ .

**Problem 37 (20 Points)**

Using resolution prove that *the Lazy Student* principle is a true statement. That is:

*There is someone in our Computational Logic class such that, if that student does the homework, then all students do the homework.*

The formal statement of the theorem may be defined as:  $\exists x (H(x) \Rightarrow \forall y H(y))$ .

**Problem 38 (30 Points)**

Consider the following facts and formalize them using first order logic. Some implicit facts need to be additionally formalized, e.g., *yellow canary is a bird and Garfield is not*.

- a) Coco is a parrot.
- b) Every parrot is a bird.
- c) No bird eats other birds.
- d) Either Coco or Garfield ate the yellow canary.

Either use resolution to prove that Garfield ate the yellow canary or give a convincing argument why this is not possible.