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Computational Logic, WS 2025/2026,
Exercise sheet 5,
due date: **23 November 2025, 23:59 via Moodle**

Problem 21 (20 Points)

Provide an example for demonstrating that the given formulas are not logically equivalent:
 $(\forall x p(x)) \Rightarrow (\forall x q(x))$ and $\forall x (p(x) \Rightarrow q(x))$.

Construct also a sequent calculus proof that the two formulas become logically equivalent if you substitute any occurrence of “forall” by “exists” and “implication” by “disjunction”.

Problem 22 (15 Points)

Consider the following sentences:

- a) *All the vaccinated cats and dogs are healthy.*
- b) *There is at least one mammal who is not healthy.*
- c) *There is at least one vaccinated mammal who is neither a cat nor a dog.*

Formalize the sentences and prove that 3. is not a logical consequence of 1. and 2.

Hint: It is sufficient to find a concrete interpretation I for which the logical consequence does not hold.

Problem 23 (25 Points)

Convert the following formulas into prenex normal form, then eliminate all existential quantifiers via skolemization:

- a) $(\forall x \exists y p(x, y)) \Rightarrow \forall x p(x, x)$
- b) $\exists z (\forall x \exists y p(x, y)) \Rightarrow \forall x p(x, z)$
- c) $\forall w \neg (\exists x \exists y \forall z p(x, z) \Rightarrow q(y, z)) \wedge \exists z p(w, z)$

Problem 24 (20 Points)

Using Sequent Calculus Trainer prove the following:

- a) $(\forall x p(x) \Rightarrow q(x)) \vdash (\exists x p(x)) \Rightarrow (\exists x q(x))$
- b) $(\exists x \forall y p(x) \vee q(y)) \vdash (\exists x p(x)) \vee (\forall y q(y))$

You need to supply screenshots integrated into the pdf file of your solution.

Problem 25 (20 Points)

Given the formula:

$$\phi = \forall x \forall y (P(f(x), g(y)) \wedge \neg P(g(x), f(y)))$$

- a) Specify a Herbrand model for ϕ .
- b) Specify a Herbrand structure suitable for ϕ , which is not a model of ϕ .