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Computational Logic, WS 2025/2026,
Exercise sheet 4,
due date: **16 November 2025, 23:59 via Moodle**

Problem 16 (15 Points)

Three students, Alice, Bob and Charlie are accused of introducing a dangerous virus at the university. During the interrogation they make the following statements:

- Alice: *Bob did it and Charlie is innocent.*
- Bob: *If Alice is guilty then Charlie is guilty too.*
- Charlie: *I did not do it. One of the others, or both of them did it.*

Using SAT solving manually (e.g., DPLL algorithm), answer the following questions:

- a) Are the three statements contradictory?
- b) Assuming that all of them are guilty, who lied during the interrogation?
- c) Assuming that nobody lied, who is innocent and who is guilty?

Problem 17 (30 Points)

Consider the following situation:

Prof. Adam gives a lecture course on Cat Psychology. He asks his five students whether everybody has a cat.

- Alice: "Noone has a cat."
- Bob: "One person has a cat."
- Charlie: "Two people have cats."
- Dora: "Three people have cats."
- Emma: "Four people have cats."

Prof. Adam knows that only those who have their own cats would be telling the truth and those who do not, would be lying.

Using Limboole, help prof. Adam finding who is telling the truth. You need to supply screenshots of your input and output files integrated into the pdf file of your solution.

Problem 18 (15 Points)

Express the following sentences in First Order Logic.

- a) Emma is a cat who does not like to swim.
- b) Terry, Berry, and Molly are all cats.
- c) Terry is larger than Berry, and Berry is larger than Molly.

- d) No cat likes to swim.
- e) Any animal that likes to swim is not a cat.
- f) There is a cat that is larger than Terry.
- g) No animal that likes to swim is larger than Berry.
- h) Any animal that does not like to swim is larger than Molly.
- i) No cat is larger than itself.

Problem 19 (20 Points)

Negate and simplify as much as possible the following formulas:

- a) $(\forall a \forall b \forall c \forall n (a > 0 \wedge b > 0 \wedge c > 0 \wedge n > 2) \Rightarrow a^n + b^n \neq c^n)$
- b) $(\forall q \exists p \forall x \forall y (p > q \wedge x > 1 \wedge y > 1) \Rightarrow x \cdot y \neq p)$

Classify all the symbols appearing in the formulas, i.e., indicate whether they are variables, constants, function symbols, predicate symbols, connectives, or quantifiers.

Compute for each formula the set of free variables of the formula (demonstrate the computation by outlining the free variables of each term and subformula).

Problem 20 (20 Points)

Let $F(x, y)$ mean that x is the father of y ; $M(x, y)$ denotes x is the mother of y . Similarly, $H(x, y)$, $S(x, y)$, and $B(x, y)$ say that x is the husband/sister/brother of y , respectively. You may also use constants to denote individuals, like “Ed”.

Translate the following sentences into predicate logic (you are not allowed to use any predicate symbols other than the above):

- a) Everybody has a mother.
- b) Everybody has a father and a mother.
- c) Whoever has a mother has a father.
- d) Ed is a grandfather.
- e) All fathers are parents.
- f) No uncle is an aunt.
- g) All brothers are siblings.