Verifying Java Programs with KeY

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Verifying Java Programs

- Extended static checking of Java programs:
  - Even if no error is reported, a program may violate its specification.
    - Unsound calculus for verifying while loops.
  - Even correct programs may trigger error reports:
    - Incomplete calculus for verifying while loops.
    - Incomplete calculus in automatic decision procedure (Simplify).
- Verification of Java programs:
  - Sound verification calculus.
    - Not unfolding of loops, but loop reasoning based on invariants.
    - Loop invariants must be typically provided by user.
  - Automatic generation of verification conditions.
    - From JML-annotated Java program, proof obligations are derived.
  - Human-guided proofs of these conditions (using a proof assistant).
    - Simple conditions automatically proved by automatic procedure.

We will now deal with an integrated environment for this purpose.

The KeY Tool

`http://www.key-project.org`

- KeY: environment for verification of JavaCard programs.
  - Subset of Java for smartcard applications and embedded systems.
  - Universities of Karlsruhe, Koblenz, Chalmers, 1998–
    - Beckert et al: "Deductive Software Verification – The KeY Book: From Theory to Practice", Springer, 2016.
    - "Chapter 16: Formal Verification with KeY: A Tutorial"
- Specification language: JML.
  - Original: OCL (Object Constraint Language), part of UML standard.
- Logical framework: Dynamic Logic (DL).
  - Successor/generalization of Hoare Logic.
  - Integrated prover with interfaces to external decision procedures.
    - Z3, CVC4, CVC5.

Now only JML is supported as a specification language.

Dynamic Logic

Further development of Hoare Logic to a modal logic.

- Hoare logic: two separate kinds of statements.
  - Formulas P, Q constraining program states.
  - Hoare triples $\{P\}C\{Q\}$ constraining state transitions.
- Dynamic logic: single kind of statement.

  Predicate logic formulas extended by two kinds of modalities.
  - $[C]Q$ $(\Leftrightarrow \neg\langle C\rangle\neg Q)$
    - Every state that can be reached by the execution of C satisfies Q.
    - The statement is trivially true, if C does not terminate.
  - $\langle C\rangle Q$ $(\Leftrightarrow \neg[C]\neg Q)$
    - There exists some state that can be reached by the execution of C and that satisfies Q.
    - The statement is only true, if C terminates.

States and state transitions can be described by DL formulas.

Dynamic Logic versus Hoare Logic

Hoare triple $\{P\}C\{Q\}$ can be expressed as a DL formula.

- Partial correctness interpretation: $P \Rightarrow [C]Q$
  - If P holds in the current state and the execution of C reaches another state, then Q holds in that state.
  - Equivalent to the partial correctness interpretation of $\{P\}C\{Q\}$.
- Total correctness interpretation: $P \Rightarrow \langle C \rangle Q$
  - If P holds in the current state, then there exists another state that can be reached by the execution of C in which Q holds.
  - If C is deterministic, there exists at most one such state; then equivalent to the total correctness interpretation of $\{P\}C\{Q\}$.

For deterministic programs, the interpretations coincide.

Advantages of Dynamic Logic

Modal formulas can also occur in the context of quantifiers.

- Hoare Logic: $\{x = a\}$ `y:=x*x` $\{x = a \land y = a^2\}$
  - Use of free mathematical variable a to denote the "old" value of x.
- Dynamic logic: $\forall a : x = a \Rightarrow [\texttt{y:=x*x}]\ x = a \land y = a^2$
  - Quantifiers can be used to restrict the scopes of mathematical variables across state transitions.

Set of DL formulas is closed under the usual logical operations.

A Calculus for Dynamic Logic

- A core language of commands (non-deterministic):

  $X := T$ . . . assignment
  $C_1; C_2$ . . . sequential composition
  $C_1 \cup C_2$ . . . non-deterministic choice
  C^* . . . iteration (zero or more times)
  $F?$ . . . test (blocks if F is false)

- A high-level language of commands (deterministic):

  skip $=$ true?
  abort $=$ false?
  $X := T$
  $C_1; C_2$
  if F **then** C_1 **else** C_2 $= (F?; C_1) \cup ((\neg F)?; C_2)$
  if F **then** C $= (F?; C) \cup (\neg F)?$
  while F **do** C $= (F?; C)^*; (\neg F)?$

A calculus is defined for dynamic logic with the core command language.

A Calculus for Dynamic Logic

- Basic rules:
  - Rules for predicate logic extended by general rules for modalities.
- Command-related rules:
  - $$\frac{\Gamma \vdash F[T/X]}{\Gamma \vdash [X := T]F}$$
  - $$\frac{\Gamma \vdash [C_1][C_2]F}{\Gamma \vdash [C_1; C_2]F}$$
  - $$\frac{\Gamma \vdash [C_1]F \qquad \Gamma \vdash [C_2]F}{\Gamma \vdash [C_1 \cup C_2]F}$$
  - $$\frac{\Gamma \vdash F \Rightarrow [C]F}{\Gamma \vdash F \Rightarrow [C^*]F}$$
  - $$\frac{\Gamma \vdash F \Rightarrow G}{\Gamma \vdash [F?]G}$$

From these, Hoare-like rules for the high-level language can be derived.

Objects and Updates

Calculus has to deal with the pointer semantics of Java objects.

- Aliasing: two variables o, o' may refer to the same object.
  - Field assignment $o.a := T$ may also affect the value of $o'.a$.
- Update formulas: $\{o.a \leftarrow T\}F$
  - Truth value of F in state after the assignment $o.a := T$.
- Field assignment rule:

$$\frac{\Gamma \ \vdash\ \{o.a \leftarrow T\}F}{\Gamma \ \vdash\ [o.a := T]F}$$

- Field access rule:

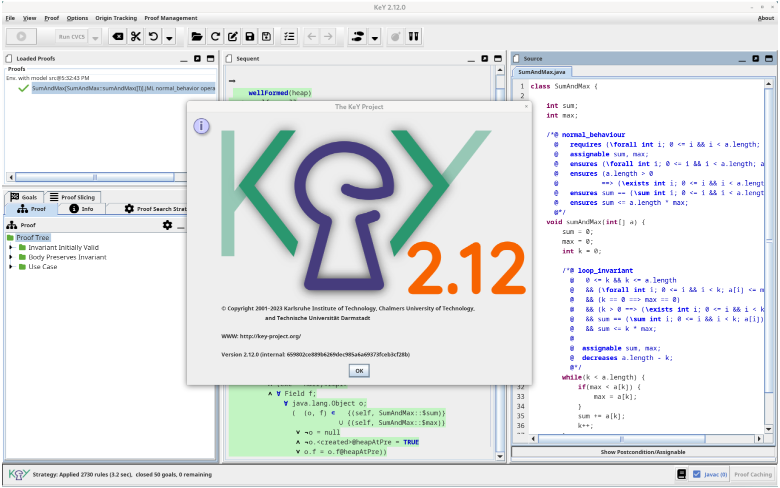
$$\frac{\Gamma, o = o' \ \vdash\ F(T) \qquad \Gamma, o \neq o' \ \vdash\ F(o'.a)}{\Gamma \ \vdash\ \{o.a \leftarrow T\}F(o'.a)}$$

  - Case distinction depending on whether o and o' refer to same object.
  - Only applied as last resort (after all other rules of the calculus).

Considerable complication of verifications.

The KeY Prover

> `> KeY &`



Env. with model src@5:32:43 PM

✓ SumAndMax[SumAndMax::sumAndMax([I)].JML normal_behavior opera

```
⇒
  wellFormed(heap)
```

```
    ∧ ∀ Field f;
        ∀ java.lang.Object o;
          (  (o, f) ∊   {(self, SumAndMax::$sum)}
                      ∪ {(self, SumAndMax::$max)}
           ∨ ¬o = null
           ∧ ¬o.<created>@heapAtPre = TRUE
           ∨ o.f = o.f@heapAtPre))
```

```
class SumAndMax {

    int sum;
    int max;

    /*@ normal_behaviour
      @   requires (\forall int i; 0 <= i && i < a.length;
      @   assignable sum, max;
      @   ensures (\forall int i; 0 <= i && i < a.length; a
      @   ensures (a.length > 0
      @           ==> (\exists int i; 0 <= i && i < a.lengt
      @   ensures sum == (\sum int i; 0 <= i && i < a.lengt
      @   ensures sum <= a.length * max;
      @*/
    void sumAndMax(int[] a) {
        sum = 0;
        max = 0;
        int k = 0;

        /*@ loop_invariant
          @   0 <= k && k <= a.length
          @   && (\forall int i; 0 <= i && i < k; a[i] <= m
          @   && (k == 0 ==> max == 0)
          @   && (k > 0 ==> (\exists int i; 0 <= i && i < k
          @   && sum == (\sum int i; 0 <= i && i < k; a[i])
          @   && sum <= k * max;
          @
          @  assignable sum, max;
          @  decreases a.length - k;
          @*/
        while(k < a.length) {
            if(max < a[k]) {
                max = a[k];
            }
            sum += a[k];
            k++;
```

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WWW: http://key-project.org/

Version 2.12.0 (internal: 659802ce889b6269dec985a6a69373fceb3cf28b)

Strategy: Applied 2730 rules (3.2 sec), closed 50 goals, 0 remaining

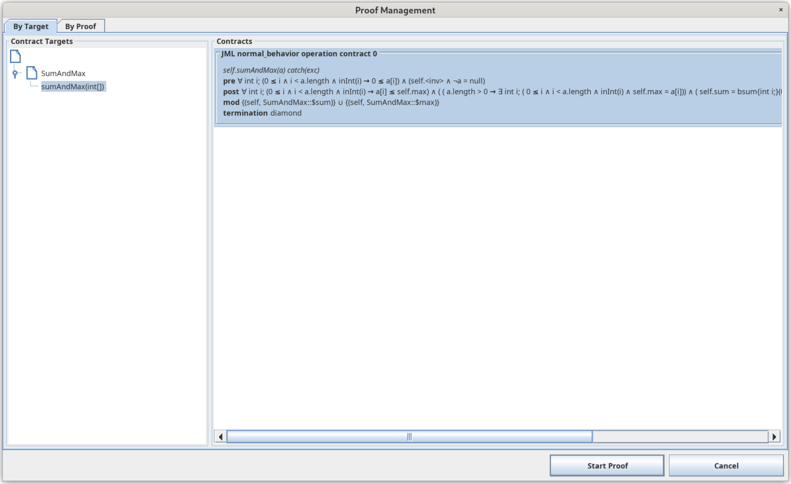
A Simple Example

File/Load Example/Getting Started/Sum and Max

```
class SumAndMax {
  int sum; int max;
  /*@ requires (\forall int i;
    @   0 <= i && i < a.length; 0 <= a[i]);
    @ assignable sum, max;
    @ ensures (\forall int i;
    @   0 <= i && i < a.length; a[i] <= max);
    @ ensures (a.length > 0 ==>
    @   (\exists int i;
    @     0 <= i && i < a.length;
    @     max == a[i]));
    @ ensures sum == (\sum int i;
    @   0 <= i && i < a.length; a[i]);
    @ ensures sum <= a.length * max;
    @*/
  void sumAndMax(int[] a) {
    sum = 0;
    max = 0;
    int k = 0;
```

```
/*@ loop_invariant
  @ 0 <= k && k <= a.length
  @ && (\forall int i;
  @      0 <= i && i < k; a[i] <= max)
  @ && (k == 0 ==> max == 0)
  @ && (k > 0 ==> (\exists int i;
  @       0 <= i && i < k; max == a[i]))
  @ && sum == (\sum int i;
  @       0 <= i && i< k; a[i])
  @ && sum <= k * max;
  @ assignable sum, max;
  @ decreases a.length - k;
  @*/
while (k < a.length) {
  if (max < a[k]) max = a[k];
  sum += a[k];
  k++;
} } }
```

A Simple Example (Contd)

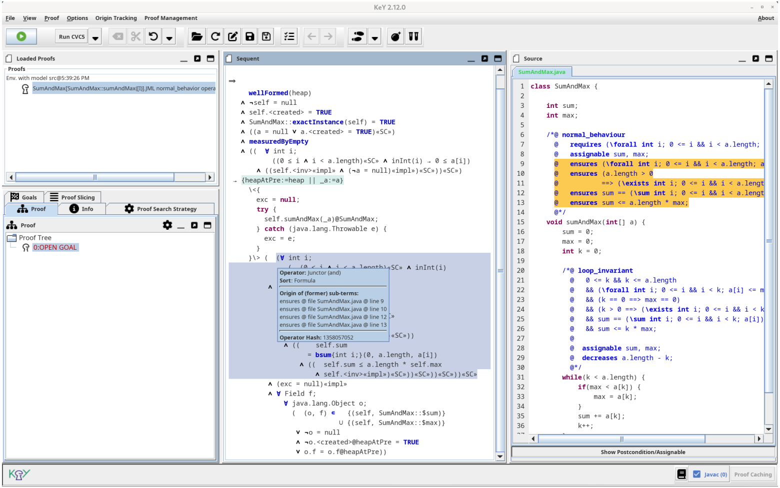


JML normal_behavior operation contract 0

self.sumAndMax(a) catch(exc)
pre ∀ int i; (0 ≤ i ∧ i < a.length ∧ inInt(i) → 0 ≤ a[i]) ∧ (self.<inv> ∧ ¬a = null)
post ∀ int i; (0 ≤ i ∧ i < a.length ∧ inInt(i) → a[i] ≤ self.max) ∧ ((a.length > 0 → ∃ int i; (0 ≤ i ∧ i < a.length ∧ inInt(i) ∧ self.max = a[i])) ∧ (self.sum = bsum{int i;}
mod {(self, SumAndMax::$sum)} ∪ {(self, SumAndMax::$max)}
termination diamond

Generate the proof obligations and choose one for verification.

A Simple Example (Contd'2)



Env. with model src@5:39:26 PM

SumAndMax[SumAndMax::sumAndMax([I])].JML normal_behavior opera

0:OPEN GOAL

```
⇒

   wellFormed(heap)
 ∧ ¬self = null
 ∧ self.<created> = TRUE
 ∧ SumAndMax::exactInstance(self) = TRUE
 ∧ ((a = null ∨ a.<created> = TRUE)«SC»)
 ∧ measuredByEmpty
 ∧ ((  ∀ int i;
          ((0 ≤ i ∧ i < a.length)«SC» ∧ inInt(i) → 0 ≤ a[i])
     ∧ ((self.<inv>«impl» ∧ (¬a = null)«impl»)«SC»))«SC»)
→ {heapAtPre:=heap || _a:=a}
   \<{
     exc = null;
     try {
       self.sumAndMax(_a)@SumAndMax;
     } catch (java.lang.Throwable e) {
       exc = e;
     }
   }\> (  (∀ int i;
            (  (0 ≤ i ∧ i < a.length)«SC» ∧ inInt(i)

       ∧
                                         »

                                         «SC»))
           ∧ ((    self.sum
                 = bsum{int i;}(0, a.length, a[i])
             ∧ ((  self.sum ≤ a.length * self.max
                 ∧ self.<inv>«impl»)«SC»))«SC»))«SC»))«SC»
       ∧ (exc = null)«impl»
       ∧ ∀ Field f;
           ∀ java.lang.Object o;
             (  (o, f) ∈   {(self, SumAndMax::$sum)}
                         ∪ {(self, SumAndMax::$max)}
              ∨ ¬o = null
              ∧ ¬o.<created>@heapAtPre = TRUE
              ∨ o.f = o.f@heapAtPre))
```

Operator: Junctor (and)
Sort: Formula

Origin of (former) sub-terms:
ensures @ file SumAndMax.java @ line 9
ensures @ file SumAndMax.java @ line 10
ensures @ file SumAndMax.java @ line 12
ensures @ file SumAndMax.java @ line 13

Operator Hash: 1358057052

```
1  class SumAndMax {
2
3      int sum;
4      int max;
5
6      /*@ normal_behaviour
7        @   requires (\forall int i; 0 <= i && i < a.length;
8        @   assignable sum, max;
9        @   ensures (\forall int i; 0 <= i && i < a.length; a
10       @   ensures (a.length > 0
11       @           ==> (\exists int i; 0 <= i && i < a.lengt
12       @   ensures sum == (\sum int i; 0 <= i && i < a.lengt
13       @   ensures sum <= a.length * max;
14       @*/
15     void sumAndMax(int[] a) {
16         sum = 0;
17         max = 0;
18         int k = 0;
19
20         /*@ loop_invariant
21           @   0 <= k && k <= a.length
22           @   && (\forall int i; 0 <= i && i < k; a[i] <= m
23           @   && (k == 0 ==> max == 0)
24           @   && (k > 0 ==> (\exists int i; 0 <= i && i < k
25           @   && sum == (\sum int i; 0 <= i && i < k; a[i])
26           @   && sum <= k * max;
27           @
28           @  assignable sum, max;
29           @  decreases a.length - k;
30           @*/
31         while(k < a.length) {
32             if(max < a[k]) {
33                 max = a[k];
34             }
35             sum += a[k];
36             k++;
```

The proof obligation in Dynamic Logic.

Proof Obligation

Two lists of formulas separated by a horizontal line.

$$\begin{array}{c} A_1 \\ \ldots \\ A_n \\ \hline B_1 \\ \ldots \\ B_m \end{array}$$

- Interpretation: $(A_1 \wedge \ldots \wedge A_n) \Rightarrow (B_1 \vee \ldots \vee B_m)$.
  - Proof is completed if some A_i is false or some B_j is true.
- All formulas are *unnegated*:
  - $(A_1 \wedge \neg A_2) \Rightarrow (B_1 \vee B_2) \rightsquigarrow A_1 \Rightarrow (B_1 \vee B_2 \vee A_2)$
  - $(A_1 \wedge A_2) \Rightarrow (B_1 \vee \neg B_2) \rightsquigarrow (A_1 \wedge A_2 \wedge B_2) \Rightarrow B_1$

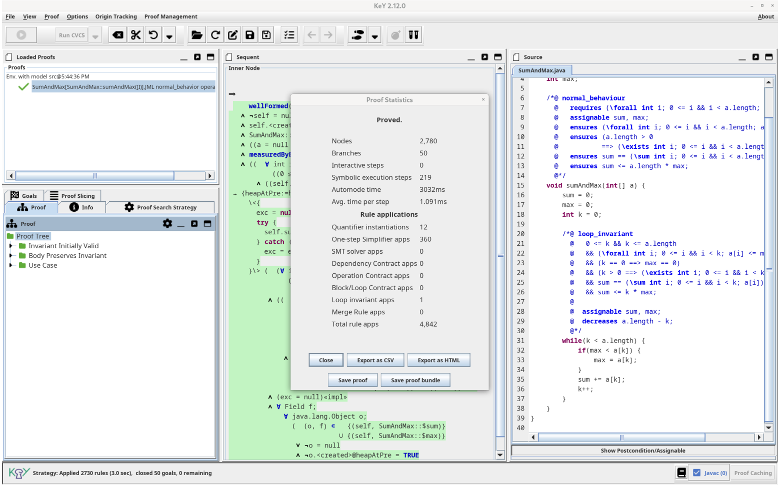
A formula below the line may represent a “negated assumption”;
a formula above the line may represent a “negated goal”:

A Simple Example (Contd'3)

```
==>
      wellFormed(heap)
    & ...
    & ((  \forall int i;
            ((0 <= i & i < a.length) & inInt(i) -> 0 <= a[i])
        & ((self_25.<inv> & (!a = null)))))
 -> {heapAtPre_0:=heap || _a:=a}
      \<{
          exc_25=null;try {
            self_25.sumAndMax(_a)@SumAndMax;
          }   catch (java.lang.Throwable e) { exc_25=e; }
        }\> (  (\forall int i;
                  (   (0 <= i & i < a.length) & inInt(i) -> a[i] <= self_25.max)
             & ((  (   a.length >  0
                    -> \exists int i;
                         ((  (0 <= i & i < a.length) & inInt(i) & self_25.max = a[i])))
                 & ((    self_25.sum = javaCastInt(bsum{int i;}(0, a.length, a[i]))
                     & ((     self_25.sum <= javaMulInt(a.length, self_25.max)
                         & self_25.<inv>)))))))
             & (exc_25 = null)
             & \forall Field f;
                 \forall java.lang.Object o;
                   (  (o, f) \in      {(self_25, SumAndMax::$sum)}
                                 \cup {(self_25, SumAndMax::$max)}
                    | !o = null
                    & !o.<created>@heapAtPre_0 = TRUE
                    | o.f = o.f@heapAtPre_0))
```

Press button "Start/stop automated proof search" (green arrow).

A Simple Example (Contd’4)



Env. with model src@5:44:36 PM

✓ SumAndMax[SumAndMax::sumAndMax([I])].JML normal_behavior opera

Proof Tree
- Invariant Initially Valid
- Body Preserves Invariant
- Use Case

```
⇒
    wellFormed(
  ∧ ¬self = nu
  ∧ self.<creat
  ∧ SumAndMax::
  ∧ ((a = null
  ∧ measuredBy
  ∧ ((  ∀ int
        ((0 ≤
      ∧ ((self.
→ {heapAtPre:=h
    \<{
      exc = nul
      try {
        self.su
      } catch (
        exc = e
      }
    }\> (  (∀ i
             (

        ∧ ((



            ∧

        ∧ (exc = null)«impl»
        ∧ ∀ Field f;
            ∀ java.lang.Object o;
              (  (o, f) ∈   {(self, SumAndMax::$sum)}
                          ∪ {(self, SumAndMax::$max)}
                ∨ ¬o = null
                ∧ ¬o.<created>@heapAtPre = TRUE
```

Proof Statistics

Proved.

Nodes	2,780
Branches	50
Interactive steps	0
Symbolic execution steps	219
Automode time	3032ms
Avg. time per step	1.091ms
Rule applications	
Quantifier instantiations	12
One-step Simplifier apps	360
SMT solver apps	0
Dependency Contract apps	0
Operation Contract apps	0
Block/Loop Contract apps	0
Loop invariant apps	1
Merge Rule apps	0
Total rule apps	4,842

Close | Export as CSV | Export as HTML | Save proof | Save proof bundle

```
    int max;

    /*@ normal_behaviour
      @   requires (\forall int i; 0 <= i && i < a.length;
      @   assignable sum, max;
      @   ensures (\forall int i; 0 <= i && i < a.length; a
      @   ensures (a.length > 0
      @           ==> (\exists int i; 0 <= i && i < a.lengt
      @   ensures sum == (\sum int i; 0 <= i && i < a.lengt
      @   ensures sum <= a.length * max;
      @*/
    void sumAndMax(int[] a) {
        sum = 0;
        max = 0;
        int k = 0;

        /*@ loop_invariant
          @   0 <= k && k <= a.length
          @   && (\forall int i; 0 <= i && i < k; a[i] <= m
          @   && (k == 0 ==> max == 0)
          @   && (k > 0 ==> (\exists int i; 0 <= i && i < k
          @   && sum == (\sum int i; 0 <= i && i < k; a[i])
          @   && sum <= k * max;
          @
          @  assignable sum, max;
          @  decreases a.length - k;
          @*/
        while(k < a.length) {
            if(max < a[k]) {
                max = a[k];
            }
            sum += a[k];
            k++;
        }
    }
}
```

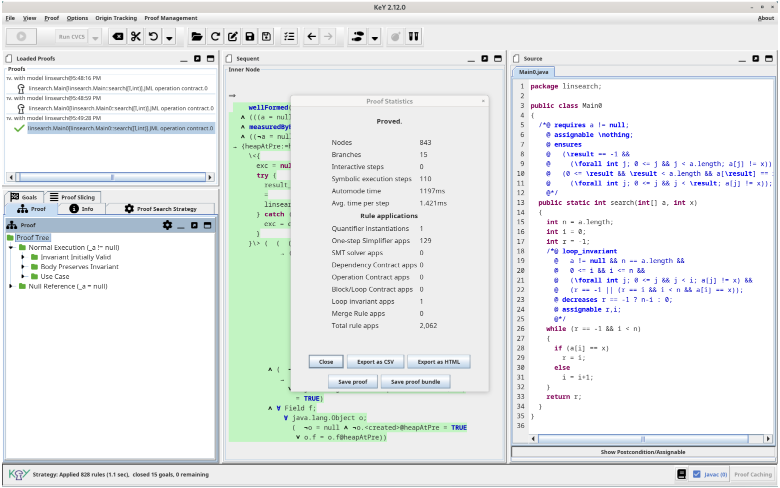
Strategy: Applied 2730 rules (3.0 sec), closed 50 goals, 0 remaining

The proof runs through automatically.

Linear Search

```
/*@ requires a != null;
  @ assignable \nothing;
  @ ensures
  @   (\result == -1 &&
  @     (\forall int j; 0 <= j && j < a.length; a[j] != x)) ||
  @   (0 <= \result && \result < a.length && a[\result] == x &&
  @     (\forall int j; 0 <= j && j < \result; a[j] != x));
  @*/
public static int search(int[] a, int x) {
  int n = a.length; int i = 0; int r = -1;
  /*@ loop_invariant
    @   a != null && n == a.length && 0 <= i && i <= n &&
    @   (\forall int j; 0 <= j && j < i; a[j] != x) &&
    @   (r == -1 || (r == i && i < n && a[r] == x));
    @ decreases r == -1 ? n-i : 0;
    @ assignable r, i; // required by KeY, not legal JML
    @*/
  while (r == -1 && i < n) {
    if (a[i] == x) r = i; else i = i+1;
  }
  return r;
}
```

Linear Search (Contd)



Proofs

ıv. with model linsearch@5:48:16 PM

linsearch.Main[linsearch.Main::search([I,int)].JML operation contract.0

ıv. with model linsearch@5:48:59 PM

linsearch.Main0[linsearch.Main0::search([I,int)].JML operation contract.0

ıv. with model linsearch@5:49:28 PM

✓ linsearch.Main0[linsearch.Main0::search([I,int)].JML operation contract.0

Proof

- Proof Tree
  - Normal Execution (_a != null)
    - Invariant Initially Valid
    - Body Preserves Invariant
    - Use Case
  - Null Reference (_a = null)

Inner Node

```
⇒
   wellFormed(
 ∧ (((a = nul
 ∧ measuredBy
 ∧ ((¬a = nul
→ {heapAtPre:=h
   \<{
     exc = nul
     try {
       result_
       =
       linsear
     } catch (
       exc = e
     }
   }\> (  (  (
          → (

        ∧ (  →
          →
          ∨
            = TRUE)
        ∧ ∀ Field f;
            ∀ java.lang.Object o;
              (  ¬o = null ∧ ¬o.<created>@heapAtPre = TRUE
               ∨ o.f = o.f@heapAtPre))
```

Proof Statistics

Proved.

Nodes	843
Branches	15
Interactive steps	0
Symbolic execution steps	110
Automode time	1197ms
Avg. time per step	1.421ms
Rule applications	
Quantifier instantiations	1
One-step Simplifier apps	129
SMT solver apps	0
Dependency Contract apps	0
Operation Contract apps	0
Block/Loop Contract apps	0
Loop invariant apps	1
Merge Rule apps	0
Total rule apps	2,062

Close | Export as CSV | Export as HTML | Save proof | Save proof bundle

Main0.java

```java
package linsearch;

public class Main0
{
  /*@ requires a != null;
    @ assignable \nothing;
    @ ensures
    @   (\result == -1 &&
    @     (\forall int j; 0 <= j && j < a.length; a[j] != x))
    @   (0 <= \result && \result < a.length && a[\result] ==
    @     (\forall int j; 0 <= j && j < \result; a[j] != x));
    @*/
  public static int search(int[] a, int x)
  {
    int n = a.length;
    int i = 0;
    int r = -1;
    /*@ loop_invariant
      @   a != null && n == a.length &&
      @   0 <= i && i <= n &&
      @   (\forall int j; 0 <= j && j < i; a[j] != x) &&
      @   (r == -1 || (r == i && i < n && a[i] == x));
      @ decreases r == -1 ? n-i : 0;
      @ assignable r,i;
      @*/
    while (r == -1 && i < n)
    {
      if (a[i] == x)
        r = i;
      else
        i = i+1;
    }
    return r;
  }
}
```

Show Postcondition/Assignable

Strategy: Applied 828 rules (1.1 sec), closed 15 goals, 0 remaining

Also this verification is completed automatically.

Proof Structure

Proof

Proof Tree

- Invariant Initially Valid
- Body Preserves Invariant
- Use Case

- Multiple conditions (Taclet option “javaLoopTreatment::teaching”):
  - Invariant Initially Valid.
  - Body Preserves Invariant.
  - Use Case (on loop exit, invariant implies postcondition).
- If proof fails, elaborate which part causes trouble and potentially correct program, specification, loop annotations.

For a successful proof, in general multiple iterations of automatic proof search (button “Start”) and invocation of separate SMT solvers required (button “Run CVC5”).

Summary

- Various academic approaches to verifying Java(Card) programs.
  - Jack: http://www-sop.inria.fr/everest/soft/Jack/jack.html
  - VeriFast: https://github.com/verifast/
  - Various tools for byte code verification.
- Do not yet scale to verification of full Java applications.
  - General language/program model is too complex.
  - Simplifying assumptions about program may be made.
  - Possibly only special properties may be verified.
- Nevertheless very helpful for reasoning on Java in the small.
  - Much beyond Hoare calculus on programs in toy languages.
  - Probably all examples in this course can be solved automatically by the use of the KeY prover and its integrated SMT solvers.
- Enforce clearer understanding of language features.
  - Perhaps constructs with complex reasoning are not a good idea...

In a not too distant future, customers might demand that some critical code is shipped with formal certificates (correctness proofs)...