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Computational Logic, WS 2024/2025,
Exercise sheet 9,
due date: **19 January 2025, 23:59 via Moodle**

Problem 39 (30 Points)

Let eq be a binary predicate such that $(\forall x\; eq(x, x))$, $(\forall x \forall y\; eq(x, y) \Rightarrow eq(y, x))$ and $(\forall x \forall y \forall z\; eq(x, y) \wedge eq(y, z) \Rightarrow eq(x, z))$. Consider the following formulas:

a) $(\exists x\; \exists y\; \exists z\; \forall t\; eq(t, x) \vee eq(t, y) \vee eq(t, z))$

b) $(\forall x\; \exists y\; \exists z\; \exists t\; \neg eq(x, y) \wedge \neg eq(x, z) \wedge \neg eq(x, t) \wedge \neg eq(y, z) \wedge \neg eq(y, t) \wedge \neg eq(z, t))$

Using the tableau procedure with free variables and unification prove that the conjunction of the two formulas is unsatisfiable.

Problem 40 (20 Points)

Let x, y, z and t be variables; a and b constants. Consider the following set of clauses:

$$\{\neg P(f(x, y), z) \vee R(z), Q(x) \vee \neg R(a), P(t, a) \vee R(a), \neg Q(b)\}$$

a) Prove by first-order resolution that this set is unsatisfiable.

b) Translate this set of clauses into both a set of Horn clauses (using the concrete syntax of facts and rules) and into a Prolog program, if possible; otherwise, explain why such translations are not possible.

Problem 41 (20 Points)

Consider the following logic program:
$r(g(X)) \leftarrow t(X, Y, f(X))$
$t(a, b, f(a)) \leftarrow$
$p(V, W) \leftarrow r(V)$
with query $\leftarrow p(U, b)$.
What will happen when using SLD resolution?

Problem 42 (30 Points)

Formalize each group of sentences, then prove that the last one is a consequence of the others using resolution and paramodulation.

a)
- Rex is a dog whose owner lives in a white house.
- Rob is the only person who lives in a white house.
- Therefore, Rob is the owner of Rex.

b)
- Rob owns only white dogs.
- Rex and Daisy have the same owner.
- Rob is the owner of Daisy.
- Therefore, Rex is a white dog.