Specifying and Verifying Programs (Part 2)

Wolfgang Schreiner
Wolfgang.Schreiner@risc.jku.at

Research Institute for Symbolic Computation (RISC)
Johannes Kepler University, Linz, Austria
http://www.risc.jku.at

1. **Programs as State Relations**

2. The RISC ProgramExplorer

Specification by State Predicates

- Hoare calculus and predicate transformers use state predicates.
  - Formulas that talk about a single (pre/post-)state.
  - In such a formula, a reference x means "the value of program variable x in the given state".
- Relationship between pre/post-state is not directly expressible.
  - Requires uninterpreted mathematical constants.

    $\{x = a\}x := x + 1\{x = a + 1\}$
- Unchanged variables have to be explicitly specified.

    $\{x = a \wedge y = b\}x := x + 1\{x = a + 1 \wedge y = b\}$
- The semantics of a command c is only implicitly specified.
  - Specifications depend on auxiliary state conditions P, Q.

    $\{P\}c\{Q\}$
    $\text{wp}(c, Q) = P$

Let us turn our focus from individual states to pairs of states.

Specification by State Relations

- We introduce formulas that denote state relations.
  - Talk about a pair of states (the pre-state and the post-state).
  - old x: "the value of program variable x in the pre-state".
  - var x: "the value of program variable x in the post-state".
- We introduce the logical judgment $c : [F]^{x,\dots}$
  - If the execution of c terminates normally, the resulting post-state is related to the pre-state as described by F.
  - Every variable y not listed in the set of variables $x, \dots$ has the same value in the pre-state and in the post-state.

$$c : F \wedge \text{var } y = \text{old } y \wedge \dots$$

$$x := x + 1 : [\text{var } x = \text{old } x + 1]^x$$
$$x := x + 1 : \text{var } x = \text{old } x + 1 \wedge \text{var } y = \text{old } y \wedge \text{var } z = \text{old } z \wedge \dots$$

We will discuss the termination of commands later.

State Relation Rules

$$\frac{c : [F]^{xs} \qquad y \notin xs}{c : [F \wedge \text{var } y = \text{old } y]^{xs \cup \{y\}}}$$

$$\textbf{skip} : [\text{true}]^{\emptyset} \qquad \textbf{abort} : [\text{true}]^{\emptyset} \qquad x = e : [\text{var } x = e']^{\{x\}}$$

$$\frac{c_1 : [F_1]^{xs} \quad c_2 : [F_2]^{xs}}{c_1 ; c_2 : [\exists ys : F_1[ys/\text{var } xs] \wedge F_2[ys/\text{old } xs]]^{xs}}$$

$$\frac{c : [F]^{xs}}{\textbf{if } e \textbf{ then } c : [\text{if } e' \text{ then } F \text{ else var } xs = \text{old } xs]^{xs}}$$

$$\frac{c_1 : [F_1]^{xs} \qquad c_2 : [F_2]^{xs}}{\textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 : [\text{if } e' \text{ then } F_1 \text{ else } F_2]^{xs}}$$

$$\frac{\begin{array}{c} c : [F]^{xs} \\ \vdash \forall xs, ys, zs : I[xs/\text{old } xs, ys/\text{var } xs] \wedge e[ys/xs] \wedge F[ys/\text{old } xs, zs/\text{var } xs] \Rightarrow \\ I[xs/\text{old } xs, zs/\text{var } xs] \end{array}}{\textbf{while } e \textbf{ do } \{I, t\} \ c : [\neg e'' \wedge (I[\text{old } xs/\text{var } xs] \Rightarrow I)]^{xs}}$$

if e then F_1 else $F_2 :\Leftrightarrow (e \Rightarrow F_1) \wedge (\neg e \Rightarrow F_2)$
$e' := e[\text{old } xs/xs], e'' := e[\text{var } xs/xs]$ (for all program variables xs)

Example

$$c_1 = y := y + 1;$$
$$c_2 = x := x + y$$

$$c_1 : [\text{var } y = \text{old } y + 1]^{y}$$
$$c_2 : [\text{var } x = \text{old } x + \text{old } y]^{x}$$

$$c_1 : [\text{var } y = \text{old } y + 1 \wedge \text{var } x = \text{old } x]^{x,y}$$
$$c_2 : [\text{var } x = \text{old } x + \text{old } y \wedge \text{var } y = \text{old } y]^{x,y}$$

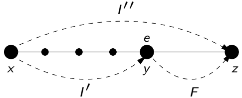
$$\begin{aligned} c_1; c_2 : [\exists x_0, y_0 : \\ & y_0 = \text{old } y + 1 \wedge x_0 = \text{old } x \wedge \\ & \text{var } x = x_0 + y_0 \wedge \text{var } y = y_0]^{x,y} \end{aligned}$$

$$c_1; c_2 : [\text{var } x = \text{old } x + \text{old } y + 1 \wedge \text{var } y = \text{old } y + 1]^{x,y}$$

Mechanical translation and logical simplification.

Loops

$$\frac{\begin{array}{c} c : [F]^{xs} \\ \vdash \forall xs, ys, zs : I[xs/\text{old } xs, ys/\text{var } xs] \land e[ys/xs] \land F[ys/\text{old } xs, zs/\text{var } xs] \Rightarrow \\ I[xs/\text{old } xs, zs/\text{var } xs] \end{array}}{\textbf{while } e \textbf{ do } \{I, t\}\ c : [\neg e'' \land (I[\text{old } xs/\text{var } xs] \Rightarrow I)]^{xs}}$$



$w = \textbf{while } i < n \textbf{ do } \{I, t\}\ (s := s + i; i := i + 1)$
$I \Leftrightarrow 0 \leq \text{var } i \leq \text{old } n \land \text{var } s = \sum_{j=0}^{\text{var } i-1} j$

$$\frac{\begin{array}{l} (s := s + i; i := i + 1) : [\text{var } s = \text{old } s + \text{old } i \land \text{var } i = \text{old } i + 1]^{s,i} \\ \vdash \forall s_x, s_y, s_z, i_x, i_y, i_z : \\ \qquad (0 \leq i_y \leq \text{old } n \land s_y = \sum_{j=0}^{i_y-1} j) \land i_y < \text{old } n \land (s_z = s_y + i_y \land i_z = i_y + 1) \Rightarrow \\ \qquad\quad 0 \leq i_z \leq \text{old } n \land s_z = \sum_{j=0}^{i_z-1} j \end{array}}{w : [\neg(\text{var } i < \text{var } n) \land (0 \leq \text{old } i \leq \text{old } n \land \text{old } s = \sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}}$$

The loop relation is derived from the invariant (not the loop body); we have to prove the preservation of the loop invariant.

Example

$c =$
$\quad$ **if** $n < 0$
$\quad\quad s := -1$
$\quad$ **else**
$\quad\quad s := 0$
$\quad\quad i := 0$
$\quad\quad$ **while** $i < n$ **do** {l,t}
$\quad\quad\quad s := s + i$
$\quad\quad\quad i := i + 1$

$I \Leftrightarrow 0 \leq \text{var } i \leq \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{var } i-1} j$
$t = \text{old } n - \text{old } i$

$c : [\text{if old } n < 0$
$\quad \text{then var } i = \text{old } i \wedge \text{var } s = -1$
$\quad \text{else var } i = \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{old } n-1} j)]^{s,i}$

Let us calculate this "semantic essence" of the program.

Example

$$c = \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b$$
$$b = (s := 0; i := 0; w)$$
$$w = \textbf{while } i < n \textbf{ do } \{I, t\}\ (s := s + i; i = i + 1)$$

$$s := 0 : [\text{var } s = 0]^{s}$$
$$s := 0 : [\text{var } s = 0 \wedge \text{var } i = \text{old } i]^{s,i}$$

$$i := 0 : [\text{var } i = 0]^{i}$$
$$i := 0 : [\text{var } i = 0 \wedge \text{var } s = \text{old } s]^{s,i}$$

$$s := 0; i := 0 : [\exists s_0, i_0 : s_0 = 0 \wedge i_0 = \text{old } i \wedge \ \text{var } i = 0 \wedge \text{var } s = s_0]^{s,i}$$
$$s := 0; i := 0 : [\text{var } s = 0 \wedge \text{var } i = 0]^{s,i}$$

$$w : [\neg(\text{var } i < \text{var } n) \wedge (0 \leq \text{old } i \leq \text{old } n \wedge \text{old } s = \textstyle\sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}$$
$$w : [\text{var } i \geq \text{old } n \wedge (0 \leq \text{old } i \leq \text{old } n \wedge \text{old } s = \textstyle\sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}$$

Example

$$
\begin{aligned}
c &= \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b\\
b &= (s := 0; i := 0; w)\\
w &= \textbf{while } i < n \textbf{ do } \{I, t\}\ (s := s + i; i = i + 1)
\end{aligned}
$$

$$
\begin{aligned}
&s := 0; i := 0 : [\text{var } s = 0 \land \text{var } i = 0]^{s,i}\\
&w : [\text{var } i \geq \text{old } n \land (0 \leq \text{old } i \leq \text{old } n \land \text{old } s = \textstyle\sum_{j=0}^{\text{old } i-1} j \Rightarrow I)]^{s,i}
\end{aligned}
$$

$$
\begin{aligned}
&b : [\exists s_0, i_0 : s_0 = 0 \land i_0 = 0 \land\\
&\qquad \text{var } i \geq \text{old } n \land (0 \leq i_0 \leq \text{old } n \land s_0 = \textstyle\sum_{j=0}^{i_0-1} j \Rightarrow I)]^{s,i}\\
&b : [\exists s_0, i_0 : s_0 = 0 \land i_0 = 0 \land\\
&\qquad \text{var } i \geq \text{old } n \land (0 \leq \text{old } n \Rightarrow I)]^{s,i}\\
&b : [\text{var } i \geq \text{old } n \land\\
&\qquad (0 \leq \text{old } n \Rightarrow 0 \leq \text{var } i \leq \text{old } n \land \text{var } s = \textstyle\sum_{j=0}^{\text{var } i-1} j)]^{s,i}\\
&b : [\text{var } i \geq \text{old } n \land\\
&\qquad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \land \text{var } s = \textstyle\sum_{j=0}^{\text{old } n-1} j)]^{s,i}
\end{aligned}
$$

Example

$$c = \textbf{if } n < 0 \textbf{ then } s := -1 \textbf{ else } b$$
$$b = (s := 0; i := 0; w)$$
$$w = \textbf{while } i < n \textbf{ do } \{I, t\}\ (s := s + i; i = i + 1)$$

$$s := -1 : [\text{var } s = -1]^{s}$$
$$s := -1 : [\text{var } i = \text{old } i \wedge \text{var } s = -1]^{s,i}$$

$$\begin{aligned} b : [&\text{var } i \geq \text{old } n \wedge \\ &\quad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \wedge \text{var } s = \textstyle\sum_{j=0}^{\text{old } n-1} j)]^{s,i} \end{aligned}$$

$$\begin{aligned} c : [&\text{if old } n < 0 \\ &\quad \text{then var } i = \text{old } i \wedge \text{var } s = -1 \\ &\quad \text{else var } i \geq \text{old } n \wedge \\ &\qquad (0 \leq \text{old } n \Rightarrow \text{var } i = \text{old } n \wedge \text{var } s = \textstyle\sum_{j=0}^{\text{old } n-1} j)]^{s,i} \end{aligned}$$

$$\begin{aligned} c : [&\text{if old } n < 0 \\ &\quad \text{then var } i = \text{old } i \wedge \text{var } s = -1 \\ &\quad \text{else var } i = \text{old } n \wedge \text{var } s = \textstyle\sum_{j=0}^{\text{old } n-1} j)]^{s,i} \end{aligned}$$

Partial Correctness

- Specification (xs, P, Q)
  - Set of program variables xs (which may be modified).
  - Precondition P (a formula with "old xs" but no "var xs").
  - Postcondition Q (a formula with both "old xs" and "var xs").
- Partial correctness of implementation c
  1. Derive $c : [F]^{xs}$.
  2. Prove $F \Rightarrow (P \Rightarrow Q)$
     - Or: $P \Rightarrow (F \Rightarrow Q)$
     - Or: $(P \wedge F) \Rightarrow Q$

Verification of partial correctness leads to the proof of an implication.

Relationship to Other Calculi

Let all state conditions refer via "*old* xs" to program variables xs.

- Hoare Calculus
  - For proving $\{P\}c\{Q\}$,
  - it suffices to derive $c : [F]^{xs}$
  - and prove $P \wedge F \Rightarrow Q[\text{var } xs/\text{old } xs]$.
- Predicate Transformers
  - Assume we can derive $c : [F]^{xs}$.
  - If c does not contain loops, then

    $$\begin{aligned}\text{wp}(c, Q) &= \forall xs : F[xs/\text{var } xs] \Rightarrow Q[xs/\text{old } xs]\\ \text{sp}(c, P) &= \exists xs : P[xs/\text{old } xs] \wedge F[xs/\text{old } xs, \text{old } xs/\text{var } xs]\end{aligned}$$
  - If c contains loops, the result is still a valid pre/post-condition but not necessarily the weakest/strongest one.

A generalization of the previously presented calculi.

Termination

- We introduce a judgment $c \downarrow T$.

  State condition T (a formula with "old xs" but no "var xs").

  - Starting with a pre-state that satisfies condition T the execution of command c terminates.

- Total correctness of implementation c.

  Specification (xs, P, Q).

  - Derive $c \downarrow T$.
  - Prove $P \Rightarrow T$.

Also verification of termination leads to the proof of an implication.

Termination Condition Rules

$$\textbf{skip} \downarrow \text{true} \qquad \textbf{abort} \downarrow \text{true} \qquad x := e \downarrow \text{true}$$

$$\frac{c_1 \downarrow T_1 \qquad c_2 \downarrow T_2}{c_1; c_2 \downarrow T_1 \wedge \text{wp}(c_1, T_2)}$$

$$\frac{c \downarrow T}{\textbf{if } e \textbf{ then } c \downarrow e' \Rightarrow T}$$

$$\frac{c_1 \downarrow T_1 \qquad c_2 \downarrow T_2}{\textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 \downarrow \text{if } e' \text{ then } T_1 \text{ else } T_2}$$

$$\frac{\begin{array}{c} c : [F]^{xs} \qquad c \downarrow T \\ \vdash \forall xs, ys, zs : \\ I[xs/\text{old } xs, ys/\text{var } xs] \wedge e[ys/xs] \wedge F[ys/\text{old } xs, zs/\text{var } xs] \wedge t[ys/\text{old } xs] \geq 0 \Rightarrow \\ T[ys/\text{old } xs] \wedge 0 \leq t[zs/\text{old } xs] < t[ys/\text{old } xs] \end{array}}{\textbf{while } e \textbf{ do } \{I, t\}\ c \downarrow t \geq 0}$$

In every iteration of a loop, the loop body must terminate and the termination term must decrease (but not become negative).

Example

$c =$

if $n < 0$
    $s := -1$
else
    $s := 0$
    $i := 0$
    while $i < n$ **do** $\{\text{I,t}\}$
        $s := s + i$
        $i := i + 1$

$I \Leftrightarrow 0 \leq \text{var } i \leq \text{old } n \wedge \text{var } s = \sum_{j=0}^{\text{var } i-1} j]$
$t = \text{old } n \; - \; \text{old } i$

$c \downarrow \text{if old } n < 0 \text{ then true else } \ldots$
$c \downarrow \text{if old } n < 0 \text{ then true else old } n \geq 0$
$c \downarrow \text{true}$

We still have to prove the constraint on the loop iteration.

Example

$$s := s + i; i := i + 1 \downarrow \text{true}$$

$$\begin{array}{l}
\forall s_x, s_y, s_z, i_x, i_y, i_z : \\
\quad (0 \leq i_y \leq \text{old } n \wedge s_y = \sum_{j=0}^{i_y - 1} j) \wedge \\
\quad i_y < \text{old } n \wedge \\
\quad (s_z = s_y + i_y \wedge i_z = i_y + 1) \wedge \\
\quad \text{old } n - i_y \geq 0 \;\Rightarrow \\
\qquad \text{true} \wedge \\
\qquad 0 \leq \text{old } n - i_z < \text{old } n - i_y
\end{array}$$

Also this constraint is simple to prove.

Abortion

Also abortion can be ruled out by proving side conditions in the usual way.

> Wolfgang Schreiner. *Computer-Assisted Program Reasoning Based on a Relational Semantics of Programs.* Research Institute for Symbolic Computation (RISC), Johannes Kepler University, Linz, Austria, 2011.

See the report for the full calculus.

1. Programs as State Relations

2. The RISC ProgramExplorer

The RISC ProgramExplorer

- An integrated environment for program reasoning.
  - Research Institute for Symbolic Computation (RISC), 2008–.
    http://www.risc.jku.at/research/formal/software/ProgramExplorer
    - Integrates the RISC ProofNavigator for computer-assisted proving.
    - Written in Java, runs under Linux (only), freely available (GPL).
- Programs written in "MiniJava".
  - Subset of Java with full support of control flow interruptions.
  - Value (not pointer) semantics for arrays and objects.
- Theories and specifications written in a formula language.
  - Derived from the language of the RISC ProofNavigator.
- Semantic analysis and verification.
  - Program methods are translated into their "semantic essence".
    - Open for human inspection.
  - From the semantics, the verification tasks are generated.
    - Solved by automatic decision procedure or interactive proof.

Tight integration of executable programs, declarative specifications, mathematical semantics, and verification tasks.

Using the Software

See "The RISC ProgramExplorer: Tutorial and Manual".

- Develop a theory.
  - File "*Theory*.theory" with a `theory` *`Theory`* of mathematical types, constants, functions, predicates, axioms, and theorems.
  - Can be also added to a program file.
- Develop a program.
  - File "*Class*.java" with a `class` *`Class`* that contains class (static) and object (non-static) variables, methods and constructors.
  - Class may be annotated by a theory (and an object invariant).
  - Methods may be annotated by method specifications.
  - Loops may be annotated by invariants and termination terms.
- Analyze method semantics.
  - Transition relations, termination conditions, ... of the method body and its individual commands.
- Perform verification tasks.
  - Frame, postcondition, termination, preconditions, loop-related tasks, type-checking conditions.

Starting the Software

- Starting the software:

  `module load ProgramExplorer` (only at RISC)

  `ProgramExplorer &`

- Command line options:

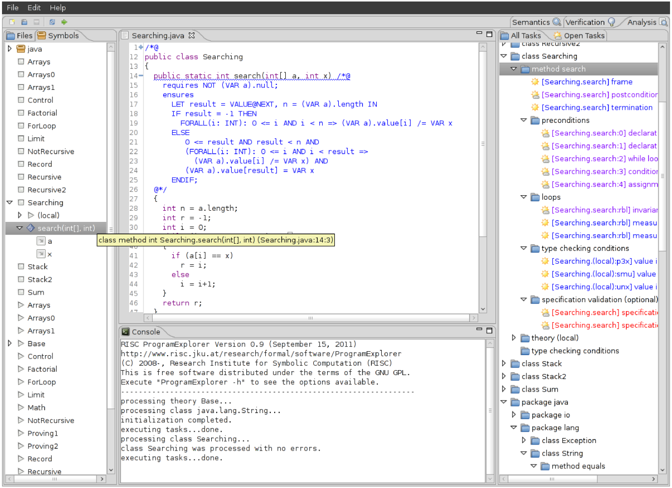
```
Usage: ProgramExplorer [OPTION]...
OPTION: one of the following options:
    -h, --help: print this message.
    -cp, --classpath [PATH]:
      directories representing top package.

Environment Variables:
    PE_CLASSPATH:
        the directories (separated by ":") representing the
        top package (default the current working directory)
    ...
```

- Task repository created/read in current working directory:

  Subdirectory `.PETASKS.`*timestamp* (ProgramExplorer tasks)

  Subdirectory `.ProofNavigator` (ProofNavigator legacy)

The Graphical User Interface



A Program

```
/*@..
class Sum
{
  static int sum(int n) /*@..
  {
    int s;
    if (n < 0)
      s = -1;
    else
    {
      s = 0;
      int i = 1;
      while (i <= n) /*@..
      {
        s = s+i;
        i = i+1;
      }
    }
    return s;
  }
}
```

Markers /*@.. indicate hidden mathematical annotations.

A Theory

```
/*@
  theory {
    sum: (INT, INT) -> INT;
    sumaxiom: AXIOM
      FORALL(m: INT, n: INT):
        IF n<m THEN
          sum(m, n) = 0
        ELSE
          sum(m, n) = n+sum(m, n-1)
        ENDIF;
  }
@*/
class Sum
...
```

The introduction of a function $sum(m, n) = \sum_{j=m}^{n} j$.

A Method Specification

```
static int sum(int n) /*@
  requires VAR n < Base.MAX_INT;
  ensures
    LET result=VALUE@NEXT IN
    IF VAR n < 0
      THEN result = -1
      ELSE result = sum(1, VAR n)
    ENDIF;
@*/
...
```

For non-negative n, a call of program method `sum(`n`)` returns $sum(1, n)$ (and does not modify any global variable).

A Loop Annotation

```
    while (i <= n) /*@
      invariant VAR n < Base.MAX_INT
            AND 1 <= VAR i AND VAR i <= VAR n+1
            AND VAR s=sum(1, VAR i-1);
      decreases VAR n - VAR i + 1;
    @*/
    {
      s = s+i;
      i = i+1;
    }
  }
```

The loop invariant and termination term (measure).

The Specification Language

Derived from the language of the RISC ProofNavigator.

- State conditions/relations, state terms.
  - State condition: method precondition (`requires`).
  - State relation: method postcondition (`ensures`), loop invariant (`invariant`).
  - State term: termination term (`decreases`).
- References to program variables.

  `OLD` x: the value of program variable x in the pre-state.

  `VAR` x: the value of program variable x in the post-state.

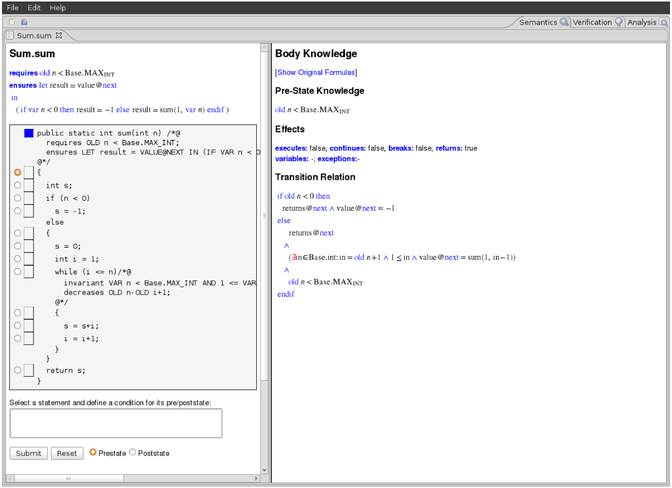
  - In state conditions/terms, both refer to the value in the current state.
  - If program variable is of the program type T, then then `OLD/VAR` x is of the mathematical type T'.

    `int` $\rightarrow$ `Base.int=[Base.MIN_INT,Base.MAX_INT]`.
- Function results

  `VALUE@NEXT`: the return value of a program function.

  - The value of the function call's post-state `NEXT`.

The Semantics View



The Method Body

Body Knowledge

[Show Original Formulas]

Pre-State Knowledge

$\text{old } n < \text{Base.MAX}_{\text{INT}}$

Effects

executes: false, **continues:** false, **breaks:** false, **returns:** true
variables: -; **exceptions:**-

Transition Relation

$$
\begin{array}{l}
\text{if old } n < 0 \text{ then} \\
\quad \text{returns@next} \wedge \text{value@next} = -1 \\
\text{else} \\
\quad \text{returns@next} \\
\quad \wedge \\
\quad (\exists \text{in} \in \text{Base.int}: \text{in} = \text{old } n+1 \wedge 1 \leq \text{in} \wedge \text{value@next} = \text{sum}(1, \text{in}-1)) \\
\quad \wedge \\
\quad \text{old } n < \text{Base.MAX}_{\text{INT}} \\
\text{endif}
\end{array}
$$

Select method symbol “sum” and menu entry ”Show Semantics”.

A Body Command

Statement Knowledge

[Show Original Formulas]

Pre-State Knowledge

$\text{old } n < \text{Base.MAX}_{\text{INT}} \land \text{old } n \geq 0 \land \text{old } s = 0 \land \text{old } i = 1$

Precondition

$\text{old } n < \text{Base.MAX}_{\text{INT}} \land 1 \leq \text{old } i \land \text{old } i \leq \text{old } n+1 \land \text{old } s = \text{sum}(1, \text{old } i-1)$

Effects

executes: true, **continues:** false, **breaks:** false, **returns:** false
variables: s, i; **exceptions:**-

Transition Relation

$\text{var } i = \text{old } n+1 \land \text{old } n < \text{Base.MAX}_{\text{INT}} \land 1 \leq \text{var } i \land \text{var } s = \text{sum}(1, \text{var } i-1)$

Termination Condition

$\text{executes@now} \Rightarrow \text{old } n - \text{old } i \geq -1$

Move the mouse pointer over the box to the left of the loop.

The Semantics Elements

- Pre-State Knowledge

  What is known about the pre-state of the command.
- Precondition

  What has to be true for the pre-state of the command such that the command may be executed.
- Effects

  Which kind of effects may the command have.
  - variables: which variables may be changed.
  - exceptions: which exceptions may be thrown.
  - executes, continues, breaks, returns: may the execution terminate normally, may it be terminated by a `continue`, `break`, `return`.
- Transition Relation

  The prestate/poststate relationship of the command.
- Termination

  What has to be true for the pre-state of the command such that the command terminates.

Formulas are shown after simplification (see "Show Original Formulas").

Constraining a State

Select the loop body, enter in the box the condition `VAR s=2 AND VAR i=1`, press “Submit”, and move the mouse to `i=i+1`.

State Conditions

[Show Original Formulas]

Pre-State Condition

$\mathrm{var}\ i = 1 \wedge \mathrm{var}\ s = \mathrm{var}\ i + 2$

Post-State Condition

$\mathrm{var}\ s = 3 \wedge \mathrm{var}\ i = 2$

The Verification Tasks

- class Sum
  - method sum
    - [Sum.sum] effects
    - [Sum.sum] postcondition
    - [Sum.sum] termination
    - preconditions
      - [Sum.sum:0] assignment precondition
      - [Sum.sum:1] while loop precondition
      - [Sum.sum:2] assignment precondition
      - [Sum.sum:3] assignment precondition
    - loops
      - [Sum.sum:qvb] invariant is preserved
      - [Sum.sum:qvb] measure is well-formed
      - [Sum.sum:qvb] measure is decreased
    - type checking conditions
    - specification validation (optional)
      - [Sum.sum] specification is satisfiable
      - [Sum.sum] specification is not trivial

The Verification Tasks

- Effects: does the method only change those global variables indicated in the method's `assignable` clause?
- Postcondition: do the method's precondition and the body's state relation imply the method's postcondition?
- Termination: does the method's precondition imply the body's termination condition?
- Precondition: does a statement's prestate knowledge imply the statement's precondition?
- Loops: is the loop invariant preserved, the measure well-formed (does not become negative) and decreased?
- Type checking conditions: are all formulas well-typed?
- Specification validation: does for every input that satisfies a precondition exist a result that does (not) satisfy the postcondition?

Partially solved by automatic decision procedure, partially by an interactive computer-supported proof.

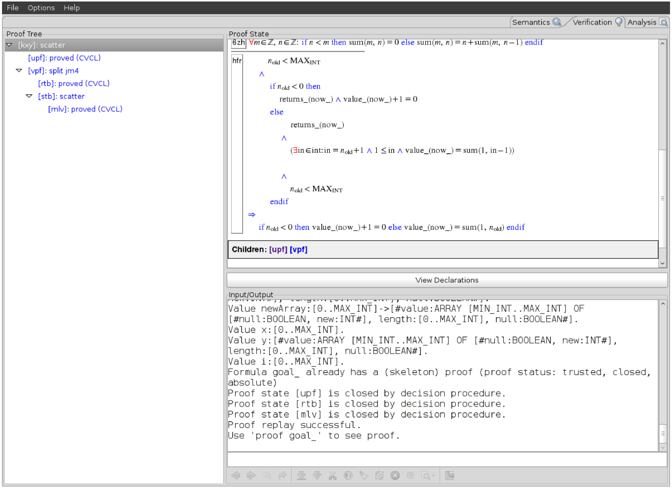
The Task States

The task status is indicated by color (icon).

- Blue (sun): the task was solved in the current execution of the RISC ProgramExplorer (automatically or by an interactive proof).
- Violet (partially clouded): the task was solved in a previous execution by an interactive proof.
  - Nothing has changed, so we need not perform the proof again.
  - However, we may replay the proof to investigate it.
- Red (partially clouded): there exists a proof but it is either not complete or cannot be trusted any more (something has changed).
- Red (fully clouded): there does not yet exist a proof.

Select "Execute Task" to start/replay a proof, "Show Proof" to display a proof, "Reset Task" to delete a proof.

A Postcondition Proof



Linear Search

```
/*@..
public class Searching
{
  public static int search(int[] a, int x) /*@..
  {
    int n = a.length;
    int r = -1;
    int i = 0;
    while (i < n && r == -1) /*@..
    {
      if (a[i] == x)
        r = i;
      else
        i = i+1;
    }
    return r;
  }
}
```

The Representation of Arrays

The program type `int[]` is mapped to the mathematical type `Base.IntArray`.

```
theory Base
{
  ...
  IntArray: TYPE =
    [#value: ARRAY int OF int, length: nat, null: BOOLEAN#];
  ...
}
```

- `(VAR` *a*`).length`: the number of elements in array *a*.
- `(VAR` *a*`).value[`*i*`]`: the element with index *i* in array *a*.
- `(VAR` *a*`).null`: *a* is the `null` pointer.

Program type *`Class`* is mapped to mathematical type *`Class`*`.`*`Class`*; *`Class`*`[]` is mapped to *`Class`*`.Array`.

Theory

```
/*@
theory uses Base {
  int: TYPE = Base.int;
  intArray: TYPE = Base.IntArray;
  smallestPosition: FORMULA
    FORALL(a: intArray, n: NAT, x: int):
      (EXISTS(i:int): 0 <= i AND i < n AND a.value[i] = x) =>
      (EXISTS(i:int): 0 <= i AND i < n AND a.value[i] = x AND
         (FORALL(j:int): 0 <= j AND j < n AND a.value[j] = x =>
           j >= i));
}
@*/
public class Searching
...
```

Method Specification

```
public static int search(int[] a, int x) /*@
  requires (VAR a).null = FALSE;
  ensures
    LET result = VALUE@NEXT, n = (VAR a).length IN
    IF result = -1 THEN
      FORALL(i: INT): 0 <= i AND i < n =>
        (VAR a).value[i] /= VAR x
    ELSE
       0 <= result AND result < n AND
       (FORALL(i: INT): 0 <= i AND i < result =>
         (VAR a).value[i] /= VAR x) AND
       (VAR a).value[result] = VAR x
    ENDIF;
@*/
...
```

Loop Annotation

```
while (i < n && r == -1) /*@
  invariant (VAR a).null = FALSE AND VAR n = (VAR a).length
        AND 0 <= VAR i AND VAR i <= VAR n
        AND (FORALL(i: INT): 0 <= i AND i < VAR i =>
              (VAR a).value[i] /= VAR x)
        AND (VAR r = -1 OR (VAR r = VAR i AND VAR i < VAR n AND
               (VAR a).value[VAR r] = VAR x));
  decreases IF VAR r = -1 THEN VAR n - VAR i ELSE 0 ENDIF;
@*/
{
  if (a[i] == x)
    r = i;
  else
    i = i+1;
}
```

Method Semantics

Transition Relation

$$(\exists \text{in} \in \text{Base.int}, n \in \text{Base.int}:$$
$$n = \text{old } a.\text{length} \wedge (\text{in} \geq n \vee \text{value@next} \neq -1) \wedge 0 \leq \text{in} \wedge \text{in} \leq n$$
$$\wedge$$
$$(\forall i \in \mathbb{Z}: 0 \leq i \wedge i < \text{in} \Rightarrow \text{old } a.\text{value}[i] \neq \text{old } x)$$
$$\wedge$$
$$(\ \text{value@next} = -1$$
$$\vee$$
$$\text{value@next} = \text{in} \wedge \text{in} < n \wedge \text{old } a.\text{value}[\text{value@next}] = \text{old } x)) \wedge \neg \text{ old } a.\text{null}$$
$$\wedge$$
$$\text{returns@next}$$

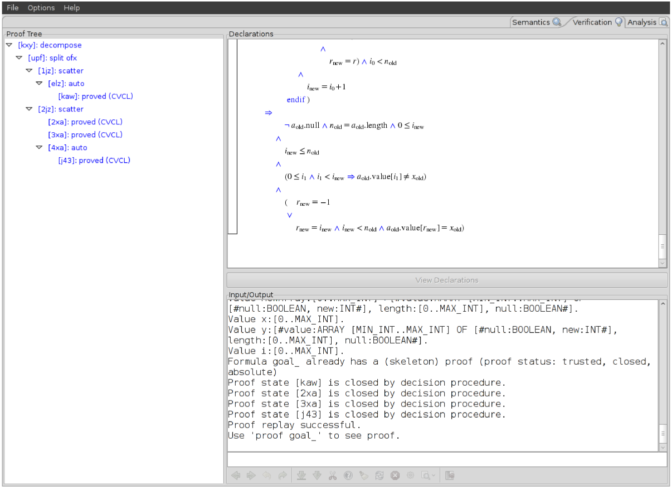
Termination Condition

$$\text{executes@now} \Rightarrow \text{old } a.\text{length} \geq 0$$

Verification Tasks

- method search
  - [Searching.search] effects
  - [Searching.search] postcondition
  - [Searching.search] termination
  - preconditions
    - [Searching.search:0] declaration precondition
    - [Searching.search:1] declaration precondition
    - [Searching.search:2] while loop precondition
    - [Searching.search:3] conditional precondition
    - [Searching.search:4] assignment precondition
  - loops
    - [Searching.search:rbl] invariant is preserved
    - [Searching.search:rbl] measure is well-formed
    - [Searching.search:rbl] measure is decreased
  - type checking conditions
    - [Searching.(local):p3x] value is in interval
    - [Searching.(local):smu] value is in interval
    - [Searching.(local):unx] value is in interval
  - specification validation (optional)

Invariant Proof



Working Strategy

- Develop theory.
  - Introduce interesting theorems that may be used in verifications.
- Develop specifications.
  - Validate specifications, e.g. by showing satisfiability and non-triviality.
- Develop program with annotations.
  - Validate programs/annotations by investigating program semantics.
- Prove postcondition and termination.
  - Partial and total correctness.
  - By proofs necessity of additional theorems may be detected.
- Prove precondition tasks and loop tasks.
  - By proofs necessity of additional theorems may be detected.
- Prove mathematical theorems.
  - Validation of auxiliary knowledge used in verifications.

The integrated development of theories, specifications, programs, annotations is crucial for the design of provably correct programs.