Computational Logic
Sample Exam Questions

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An exam has (100P) in total; the following questions amount to more than (100P).

1. (30P) Consider the following propositional formula $F$:

   $\neg(p \lor (q \land (r \lor s \Rightarrow p)))$

   a) (6P) Give the NNF of $F$.
   b) (6P) Construct the truth table for $F$ (it is not necessary to show the truth values of all subformulas).
   c) (6P) Determine from the truth table the DNF and the CNF of $F$.
   d) (12P) Derive the DNF and the CNF of $F$ by logical equivalence transformations (show the main steps).

2. (24P) Consider the following propositional formula $F$:

   $((p \lor q) \land (\neg r \Rightarrow \neg p)) \Rightarrow (r \lor q)$

   a) (6P) Prove the validity of $F$ by a sequent calculus proof;
   b) (6P) Give the CNF of the negation of $F$.
   c) (6P) Prove the validity of $F$ by a resolution proof.
   d) (6P) Prove the validity of $F$ by applying the recursive DPLL algorithm (sketch the corresponding deduction tree).

3. (18P) Consider the following first-order formula $F$:

   $\neg(\forall x. p(x) \Rightarrow ((\forall y. r(x,y)) \lor (\exists y. q(x,y))))$

   a) (6P) Give the NNF of $F$.  

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b) (6P) Give the PNF of $F$.

c) (6P) Give a formula $F'$ in SNF that is equisatisfiable with $F$.

4. (15P) Consider the following first-order formula $F$:

$$(p(c) \land \forall x. p(x) \Rightarrow q(x, f(x))) \Rightarrow (\exists y. q(c, y))$$

Show the validity of $F$ by applying the Gilmore algorithm.

5. (28P) Consider the following first-order formula $F$:

$$((\forall x. p(x) \Rightarrow q(x, f(x))) \land (\exists x. (\forall y. \neg q(x, y)))) \Rightarrow (\exists x. \neg p(x))$$

a) (8P) Prove the validity of $F$ by a sequent calculus proof.

b) (8P) Prove the validity of $F$ by the method of analytic tableaux (either the basic method or the free-variable method; indicate which variant you use).

c) (12P) Prove the validity of $F$ by the resolution method.

6. (25P) Consider the following formula $F$ in first-order logic with equality:

$$(\forall x, y. e \circ x = x \land f(e) = e \land f(x \circ y) = f(y) \circ f(x)) \Rightarrow f(a \circ (b \circ e)) = f(b) \circ f(a)$$

a) (10P) Prove the validity of $F$ by the method of analytic tableaux (either the basic method or the free-variable method; indicate which variant you use).

b) (15P) Prove the validity of $F$ by paramodulation.

7. (10P) Consider the term rewriting system $R$ induced by the following equations:

$$(x/y) * z = (x * z)/y \quad (x/y) * y = x \quad (x/x) = 1$$

a) (5P) Give the set of critical pairs of $R$.

b) (5P) Is $R$ confluent? If not, add rewrite rules that make it confluent.

8. (10P) Consider the following formula $F$ in theory LRA:

$$x \geq 1 \land 2x + 4y \leq 14 \land x - 2y \leq -1$$

Decide by the Fourier-Motzkin algorithm whether $F$ is satisfiable (show the main steps). If the answer is positive, give a satisfying assignment for $x$ and $y$.

9. (10P) Consider the following formula $F$ in theory EUF:

$$a \leq b \land b = c \land g(f(a), b) = g(f(c), a) \Rightarrow f(a) = b$$

Decide by the congruence closure algorithm whether $F$ is valid (show the main steps).

10. (10P) Consider the following formula $F$ in a combination of theories LRA and EUF:

$$a \leq b \land b \leq a \land g(a, b) = f(a) + f(b) \Rightarrow g(a, a) = 2 \cdot f(a)$$

Decide by the Nelson-Oppen Method the validity of $F$ (sketch the main steps).