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Computational Logic, WS 2023/2024,
Exercise sheet 4,
due date: **19 November 2023, 23:59 via Moodle**

Problem 16 (15 Points)
The following four facts are known:

a) The diamond has been stolen!

b) If the diamond has been stolen, a thief got in the house.

c) If a thief got in, either the door was open or the window was smashed.

d) The window is not smashed.

By means of SAT solving (via the recursive DPLL function), show manually that from this information it follows that *The door was open.*

Problem 17 (30 Points)
In chess, a queen can move as far as she pleases, horizontally, vertically, or diagonally. A chess board has 8 rows and 8 columns. The standard 8 by 8 Queen's problem asks how to place 8 queens on an ordinary chess board so that none of them can hit any other in one move. A modification of the 8 by 8 problem is to consider an N by N chess board and ask if one can place N queens on such a board.
Is there a solution to N queens puzzle for $N = 4$? If yes, draw a solution.
Using Limboole, show that there is no solution for $N = 3$. You need to supply screenshots of your input and output files integrated into the pdf file of your solution.

Problem 18 (15 Points)
Express the following sentences in First Order Logic.

a) Rex is a dog who likes to swim.

b) Rex, Johne, and Teddy are all dogs.

c) Rex is larger than Johne, and Teddy is larger than Rex.

d) All dogs like to swim.

e) Every animal that likes to swim is a dog.

f) There is a dog that is larger than Teddy.

g) No animal that likes to swim is larger than Rex.

h) Any animal that does not like to swim is larger than Johne.

i) No dog is larger than itself.

Problem 19 (20 Points)
Negate and simplify as much as possible the following formulas:

a) $\forall x \exists \epsilon((x > 0 \wedge \epsilon > 0) \wedge \forall y(y > 0 \Rightarrow x - y > \epsilon))$

b) $((\exists x \forall y Q(x, y)) \wedge \forall x(Q(x, x) \Rightarrow \exists y R(y, x))) \Rightarrow \exists y \exists x R(x, y)$

Classify all the symbols appearing in the formulas, i.e., indicate whether they are variables, constants, function symbols, predicate symbols, connectives, or quantifiers.
Compute for each formula the set of free variables of the formula (demonstrate the computation by outlining the free variables of each term and subformula).

Problem 20 (20 Points)
Let $F(x, y)$ mean that x is the father of y; $M(x, y)$ denotes x is the mother of y. Similarly, $H(x, y)$, $S(x, y)$, and $B(x, y)$ say that x is the husband/sister/brother of y, respectively. You may also use constants to denote individuals, like "Ed". However, you are not allowed to use any predicate symbols other than the above to translate the following sentences into predicate logic:

a) Everybody has a mother.

b) Everybody has a father and a mother.

c) Whoever has a mother has a father.

d) Ed is a grandfather.

e) All fathers are parents.

f) No uncle is an aunt.

g) All brothers are siblings.