

Computational Logic

Sample Exam Questions

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An exam has (100P) in total; the following questions amount to more than (100P).

1. (30P) Consider the following propositional formula F :

$$\neg(p \vee (q \wedge (r \vee s \Rightarrow p)))$$

- (6P) Give the NNF of F .
 - (6P) Construct the truth table for F (it is not necessary to show the truth values of all subformulas).
 - (6P) Determine from the truth table the DNF and the CNF of F .
 - (12P) Derive the DNF and the CNF of F by logical equivalence transformations (show the main steps).
2. (24P) Consider the following propositional formula F :

$$((p \vee q) \wedge (\neg r \Rightarrow \neg p)) \Rightarrow (r \vee q)$$

- (6P) Prove the validity of F by a sequent calculus proof;
 - (6P) Give the CNF of the negation of F .
 - (6P) Prove the validity of F by a resolution proof.
 - (6P) Prove the validity of F by applying the recursive DPLL algorithm (sketch the corresponding deduction tree).
3. (18P) Consider the following first-order formula F :

$$\neg(\forall x. p(x) \Rightarrow ((\forall y. r(x, y)) \vee (\exists y. q(x, y))))$$

- (6P) Give the NNF of F .

- b) (6P) Give the PNF of F .
- c) (6P) Give a formula F' in SNF that is equisatisfiable with F .

4. (15P) Consider the following first-order formula F :

$$(p(c) \wedge \forall x.p(x) \Rightarrow q(x, f(x))) \Rightarrow (\exists y. q(c, y))$$

Show the validity of F by applying the Gilmore algorithm.

5. (28P) Consider the following first-order formula F :

$$((\forall x. p(x) \Rightarrow q(x, f(x))) \wedge (\exists x. (\forall y. \neg q(x, y)))) \Rightarrow (\exists x. \neg p(x))$$

- a) (8P) Prove the validity of F by a sequent calculus proof.
- b) (8P) Prove the validity of F by the method of analytic tableaux (either the basic method or the free-variable method; indicate which variant you use).
- c) (12P) Prove the validity of F by the resolution method.

6. (25P) Consider the following formula F in first-order logic with equality:

$$(\forall x, y. e \circ x = x \wedge f(e) = e \wedge f(x \circ y) = f(y) \circ f(x)) \Rightarrow f(a \circ (b \circ e)) = f(b) \circ f(a)$$

- a) (10P) Prove the validity of F by the method of analytic tableaux (either the basic method or the free-variable method; indicate which variant you use).
- b) (15P) Prove the validity of F by paramodulation.

7. (10P) Consider the term rewriting system R induced by the following equations:

$$(x/y) * z = (x * z)/y \qquad (x/y) * y = x \qquad (x/x) = 1$$

- a) (5P) Give the set of critical pairs of R .
- b) (5P) Is R confluent? If not, add rewrite rules that make it confluent.

8. (10P) Consider the following formula F in theory LRA:

$$x \geq 1 \wedge 2x + 4y \leq 14 \wedge x - 2y \leq -1$$

Decide by the Fourier-Motzkin algorithm whether F is satisfiable (show the main steps). If the answer is positive, give a satisfying assignment for x and y .

9. (10P) Consider the following formula F in theory EUF:

$$a = b \wedge b = c \wedge g(f(a), b) = g(f(c), a) \Rightarrow f(a) = b$$

Decide by the congruence closure algorithm whether F is valid (show the main steps).

10. (10P) Consider the following formula F in a combination of theories LRA and EUF:

$$a \leq b \wedge b \leq a \wedge g(a, b) = f(a) + f(b) \Rightarrow g(a, a) = 2 \cdot f(a)$$

Decide by the Nelson-Oppen Method the validity of F (sketch the main steps).