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Klausur 1

Berechenbarkeit und Komplexität

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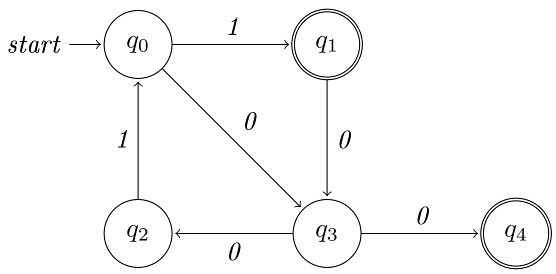
Part 1 *NFSM2012*

Let N be the nondeterministic finite state machine

$$(\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \nu, \{q_0\}, \{q_1, q_4\}),$$

whose transition function ν is given below.



1 | | no — *Is* $10010011001101 \in L(N)$*?*

A word $w \in L(N)$ with $|w| > 1$ ends either with 11 or 00, but never with 01.

2 | yes | — *Is* $001100 \in L(N)$*?*

Follow the states $q_0, q_3, q_2, q_0, q_1, q_2, q_4$.

3 | | no — *Is $L(N)$ finite?*

4 | yes | — *Is $L(N) = L(r)$ for the regular expression $r = ((\varepsilon + 1)001)^*(1 + 00 + 100)$?*

5 | yes | — *Is $\overline{L(N)}$ recursive?*

6 | yes | — *Is there a deterministic finite state machine M with less than 100 states such that $L(M) = L(N)$?*

According to the subset construction, there must be a DFSM with at most $2^5 = 32$ states.

7 | yes | — *Is there a nondeterministic Turing machine $T = (Q, \Gamma, \sqcup, \{0, 1\}, \delta, q, F)$ with less than 10 states such that $L(T) = L(N)$?*

In fact, 7 states are sufficient. We choose $\Gamma = \{0, 1, \sqcup\}$, $Q = \{q_0, \dots, q_6\}$. $p = q_0$, $F = q_6$. The transition function is (more or less) given by ν above. In each step T reads and writes the same character and always moves to the right. If T reads a blank in any of the accepting states $\{q_1, q_4\}$ of the NFSM N, it goes into the accepting state q_6 of T, if T reads a blank when in one of the states $\{q_0, q_2, q_3\}$ it goes into the non-exepting state q_5. The Turing machine T stops in states q_5 and q_6.

8 | yes | — *Does there exists a deterministic finite state machine D such that $L(D) = L(N) \circ \overline{L(N)}$?*

$L(N)$ and $\overline{L(N)}$ are both regular. Concatenation of two regular languages gives a regular language.

Part 2 Pumping2012

Let

$$L_1 = \left\{ a^m b^n a^{2m} \,\middle|\, m, n \in \mathbb{N}, m < 1000 \right\},$$
$$L_2 = \left\{ a^m b^n a^{2m} \,\middle|\, m, n \in \mathbb{N}, n < 1000 \right\}.$$

9	yes	

Is there a regular expression r such that $L(r) = L_1$?

$r = b^* + ab^*aa + aab^*aaaa + \cdots + a^{999}b^*a^{1998}$

10		no

Is there a deterministic finite state machine M such that $L(M) = \{a, b\}^* \setminus L_2$?

L_2 is not regular, i.e., its complement $\overline{L_2}$ is not regular, either.

11	yes	
12	yes	
13	yes	

11: *Is there an enumerator Turing machine G such that $Gen(G) = L_1$?*

12: *Is there an Turing machine M such that $L(M) = L_1 \cup L_2$?*

13: *Is there an deterministic finite state machine D such that $L(D) = L_1 \cap L_2$?*

The language $L_1 \cap L_2$ is finite and thus regular.

Part 3 RecursiveEnumerable6

Let M be a Turing machine with the following property: Whenever M accepts a word, it does so in no more than 1000 steps.

14	yes	

Is $L(M)$ necessarily recursive?

Start M with input w und execute 1000 steps. If w has been accepted then $w \in L(M)$, otherwise $w \notin L(M)$. Therefore, $L(M)$ and $\overline{L(M)}$ are both recursively enumerable.

15		no

Is $L(M)$ necessarily finite?

Let $M = (\{q_0\}, \{0, 1, \sqcup\}, \sqcup, \{0, 1\}, \delta, q_0, \{q_0\})$ where δ is nowhere defined. Then $L(M) = \Sigma^*$.

16		no

Let L be a recursively enumerable language. Can it be concluded that $L(M) \cap L$ is recursive?

If M is a Turing machine that accepts everything without any computation, then $L(M) = \Sigma^*$ and thus $L(M) \cap L = L$. Thus, if the intersection $L(M) \cap L$ were recursive, it would mean that every recursively enumerable language is recursive. This is clearly not the case.

Part 4 TM2012

Let $M = (Q, \Gamma, \sqcup, \Sigma, \delta, q_0, F)$ be a Turing machine with $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$, $F = \{q_2\}$. The transition function

$$\delta : Q \times \Gamma \to_P Q \times \Gamma \times \{L, R\}$$

is given by the following table.

δ	0	1	$\sqcup$
q_0	$(q_1, 0, R)$	$(q_0, 0, R)$	$(q_2, 1, R)$
q_1	$(q_0, 1, R)$	$(q_1, 0, L)$	–
q_2	–	–	–

Furthermore, let $M' = (Q, \Gamma, \sqcup, \Sigma, \delta', q_0, F)$ where δ' is (nearly) identical to δ except for the fact that $\delta'(q_1, 1)$ is undefined, i. e., δ' is given by the followig *table.*

δ'	0	1	$\sqcup$
q_0	$(q_1, 0, R)$	$(q_0, 0, R)$	$(q_2, 1, R)$
q_1	$(q_0, 1, R)$	$-$	$-$
q_2	$-$	$-$	$-$

17	yes	

Is $q_0011 \vdash 0q_111 \vdash q_1001 \vdash 1q_001 \vdash 10q_11 \vdash 1q_100 \vdash 11q_00 \vdash 110q_1\sqcup$ *a computation of* M*?*

18		no

Is $011 \in L(M)$*?*

The machine M terminates in the non-accepting state q_1.

19	yes	

Is $L(M)$ *a recursively enumerable language?*

20		no

Is $1101 \in L(M')$*?*

21		no

Is $L(M')$ *a finite set?*

22		no

Is there a word $w \in \Sigma^*$ *for which* M *does not terminate?*

The machine goes into state q_1 only after it has read a 0 and after moving the head right. Thus, if the machine sees a 1 under the head in state q_1, it is clear that the character left of this 1 is a 0, i.e. the head will only be moved at most one position to the left and then never come back to the 1 that it writes in state q_1. Eventually, the head will arrive at a blank and thus the machine stops.

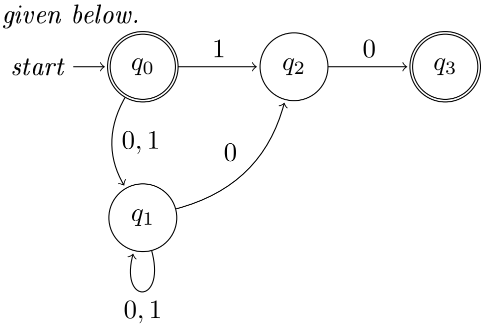
23	yes	

Is $L(M')$ *a recursive language?*

Obviously, the head is only moved to the right, so eventually, the head will be over a blank and thus the machine terminates.

Part 5 *Open2012*

((2 points))
Let $N = (Q, \Sigma, \delta, q_0, F)$ *be a nondeterministic finite state machine with* $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $S = \{q_0\}$, $F = \{q_0, q_3\}$, *and transition function* δ *as given below.*



1. *Let* X_i *denote the regular expression for the language accepted by* N *when starting in state* q_i.

   Write down an equation system for $X_0, \ldots, X_3$.

2. *Give a regular expression* r *such that* $L(r) = L(N)$ *(you may apply Arden's Lemma to the result of 1).*

$$
\begin{aligned}
X_0 &= (0+1)X_1 + 1X_2 + \varepsilon \\
X_1 &= (0+1)X_1 + 0X_2 \\
X_2 &= 0X_3 \\
X_3 &= \varepsilon \\
r &= \varepsilon + 10 + (0+1)(0+1)^*00
\end{aligned}
$$