**Problems Solved:**

| 21 | 22 | 23 | 24 | 25 |
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**Name:**

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**Problem 21.** Write a RAM program that computes the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = \sum_{k=1}^{n} k$ such that the program reads an integer $n$ from the input tape and (upon termination) has written $f(n)$ to its output tape.
Note that it is not specified what the RAM should do if it reads a negative integer. Your RAM program can thus assume that the input is non-negative.

**Problem 22.** Let $\Sigma = \{a, b\}$. We code $a$ and $b$ on the input tape of a RAM by 1 and 2 and a word $w \in \Sigma^*$ by a respective sequence of 1's and 2's.
We say that a RAM $R$ *accepts* a word $w \in \Sigma^*$ if $R$ starts with the coded word $w$ on its input tape and terminates after having written a non-zero number on its output tape. We define $L(R) := \{w \in \Sigma^* \mid R \text{ accepts } w\}$.
Let $F$ be a RAM that terminates for every input and whose program does not contain "loops", i.e., each instruction is executed at most once.
Derive answers for the following questions. (Give ample justifications, just saying 'yes' or 'no' is not enough.)

1. Is $L(F)$ as a language over $\Sigma$ finite?
2. Is $L(F)$ as a language over $\Sigma$ regular?

**Problem 23.** Give reasons for your answers.

1. Let $R$ be a RAM that reads exactly one number from its input tape and always terminates with 0 or 1 written on its output tape. Is there a function $f : \mathbb{Z} \to \mathbb{Z}$ such that $f(x) = y$ if and only if the input was $x$ and after termination $y$ is on the output tape of $R$?
2. Let $f : \mathbb{Z} \to \mathbb{Z}$ be a function. Is there always a RAM $R$ such that $R$ terminates on every input and that $R$ with input $z \in \mathbb{Z}$ has written $f(z)$ to its output tape?

**Problem 24.**

**Definition 1** (RAM computable)**.** We say that a partial function $f : \mathbb{N} \to_P \mathbb{N}$ is *RAM computable* if there exists a RAM $R$ that such that

- $R$ terminates for input $n \in \mathbb{N}$ if and only if $n \in domain(f)$;
- $R$ terminates for input $n \in \mathbb{N}$ with output $n'$ if and only if $n' = f(n)$.

Show that every loop computable function is also RAM computable.

**Problem 25.** Provide a loop program that computes the function $f(n) = \sum_{k=1}^{n} k(k+1)$, and thus show that $f$ is loop computable.