Problems Solved:

16	17	18	19	20

Name:

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Problem 16. Find a Turing machine $M = (Q, \Gamma, \sqcup, \{0, 1\}, \delta, q_0, F)$ such that $L(M) = \{1^k 0 1^{k+1} \mid k \in \mathbb{N}\}$. Write down Q, Γ, F and δ explicitly.

Problem 17. Let $M = (Q, \Gamma, \sqcup, \Sigma, \delta, q_0, F)$ be a Turing machine with $Q = \{q_0, \ldots, q_6\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$, $F = \{q_3\}$ and the following transition function δ.

δ	0	1	$\sqcup$
q_0	$q_1 0 R$	$q_4 \sqcup R$	$-$
q_1	$-$	$q_2 1 R$	$-$
q_2	$-$	$-$	$q_3 \sqcup R$
q_3	$-$	$-$	$-$
q_4	$q_4 0 R$	$q_4 1 R$	$q_5 \sqcup L$
q_5	$-$	$q_6 \sqcup L$	$-$
q_6	$q_6 0 L$	$q_6 1 L$	$q_0 \sqcup R$

Determine the set $L(M)$ without referring to M.

Problem 18. Write down explicitly a Turing machine M over $\Sigma = \{0\}$ which computes the function $d : \mathbb{N} \to \mathbb{N}$ given by $d(n) = 2n$.
Use unary representation: A number n is represented by the string 0^n consisting of n copies of the symbol 0.

Problem 19. Write down explicitly an enumerator G such that $\mathrm{Gen}(G) = \{0^{2n} \mid n \in \mathbb{N}\}$.
Since in the lecture notes it has not been *formally* defined, how a Turing machine with two tapes works, you may describe the transition function as

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, L\} \times (\Gamma \cup \{\boxtimes\})$$

in the following way: If G is in state q and reads the symbol c from the working tape, and

$$\delta(q, c) = (q', c', d, c'')$$

then G goes to state q', replaces c by c' on the working tape and moves the working tape head in direction d. Moreover, unless $c'' = \boxtimes$, the symbol c'' is written on the output tape and the output tape head is moves one position forward. If, however, $c'' = \boxtimes$, nothing is written on the output tape and the output tape head rests in place.
Hint: There exists a solution with only 4 states.

Problem 20. Show that the language $L = \{a^m b^n c^{m+n} \mid m, n \in \mathbb{N}\}$ over the alphabet $\Sigma = \{a, b, c\}$ is not regular.