Gelöste Aufgaben:

1	2	3	4	5

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Aufgabe 1. A function f is given by $f(0) = 1$ und $f(n+1) = (n+1) \cdot f(n)$ for $n \geq 0$. Show by induction that $f(2n)/f(n)^2 \leq 4^n$ for all natural numbers n.

Aufgabe 2. Show

$$\forall x \in \mathbb{R}\ \forall y \in \mathbb{R} : \quad x^2 + y^4 = 10 \implies x \leq 4.$$

Hint: (indirect proof) To show $A \implies B$ one shows that $A \wedge \neg B$ leads to a contradiction.

Aufgabe 3. Let $f_1, f_2, f_3, \ldots$ be a sequence of sequences of natural numbers. We write $f_i(n)$ for the n-th number in the i-th sequence. Let g be the sequence given by $g(n) = f_n(n) + 1$. Show

$$\forall i \in \mathbb{N} : \quad g \neq f_i.$$

In other words, show that g is not among the $f_1, f_2, f_3, \ldots$. *Hint:* (indirect proof) Assume that there were some $k \in \mathbb{N}$ such that $g = f_k$ and show that the computation of $g(k)$ leads to a contradiction.

Aufgabe 4. Let $L \subseteq \Sigma^*$ be a language over the alphabet $\Sigma = \{a, b, c, d\}$ such that a word w is in L if and only if it is either a or b or of the form $w = ducvd$ where u and v are words of L. For example, $dacad$, $ddacbdcad$, $dddbcbdcdbcbddcad$ are words in L. Show by induction that every word of L contains an even number of the letter d.
Note that a *language* is just a set of words and a *word* is simply a sequence of letters from the alphabet.

Aufgabe 5. Solve the following tasks.

1. Write down a deterministic finite state machine D whose automata language is $L(D) = \{\texttt{finite}, \texttt{language}\}$.
2. Let $L = \{10^n1 \mid n \text{ is an even number less than } 10\}$. Construct a DFSM D such that $L = L(D)$.
3. Does for each finite language L exist a DFSM M so that $L = L(M)$?