

# Formal Methods in Software Development

## Sample Exam

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**Last Name:**

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**Studienkennzahl:**

**100 points total**

1. (25P)

a) (12P) Write a RISCAL specification (pre/post-condition) of a procedure

```
val N:Nat; type int = Int[-N,N]; type array = Array[N,int];  
proc fill(a:array, p:int, n:int, e:int): array { ... }
```

which returns a copy of  $a$  where, starting from position  $p$ ,  $n$  elements have been set to  $e$ ; do not forget to specify suitable preconditions for  $p$  and  $n$  that restrict their range to reasonable limits.

b) (13P) Write a *heavy-weight* JML specification for the following method of the Java library (the specification shall be as expressive as possible).

```
public static void fill(int[] a, int fromIndex, int toIndex, int val)
```

Assigns the specified int value to each element of the specified range of the specified array of ints. The range to be filled extends from index `fromIndex`, inclusive, to index `toIndex`, exclusive. (If `fromIndex==toIndex`, the range to be filled is empty.)

Parameters:

`a` - the array to be filled  
`fromIndex` - the index of the first element (inclusive) to be filled with the specified value  
`toIndex` - the index of the last element (exclusive) to be filled with the specified value  
`val` - the value to be stored in all elements of the array

Throws:

`IllegalArgumentException` - if `fromIndex > toIndex`  
`ArrayIndexOutOfBoundsException` - if `fromIndex < 0` or `toIndex > a.length`

The stated exceptions are runtime exceptions and thus not declared in the header of the method; nevertheless, specify the situations where they are thrown.

2. (25P)

a) (13P) Derive the weakest precondition of the command  $c$

```
if (i < 10) {  
    a[i] = a[i]+3;  
    i = i+1;  
}
```

for postcondition  $a[2] = 5$  (ignoring ‘index out of bound’ violations). Simplify the derived precondition as far as possible.

b) (12P) Derive the strongest postcondition of above command for precondition  $a[2] = 5$  and simplify it as far as possible.

Remember (for both parts) that an array assignment  $a[i] := b$  is just an abbreviation for the scalar assignment  $a := a[i \mapsto b]$ .

3. (25P) Take the following program which is supposed to compute for given  $n \in \mathbb{N}$  the result  $s := n^2$ :

```

{n = oldn}

s = 0; i = 1;
while (i <= n)
{
    s = s+2*i-1;
    i = i+1;
}

```

$\{s = n^2 \wedge n = oldn\}$

- a) (13P) Assume you are given a suitable loop invariant  $I$  and termination term  $T$ ; using  $I$  and  $T$  state all verification conditions (classical logic formulas) that have to be proved for verifying partial correctness and termination of the program (writing  $I[t/x]$  for a substitution of term  $t$  for variable  $x$  in  $I$  and analogously  $T[t/x]$ ).
- b) (12P) Construct for input  $n = 10$  a table for the values of the variables before/after each loop iteration. Using this table as a hint, give suitable definitions for  $I$  and  $T$ . Demonstrate how from your choice of  $I$  it can be concluded that the invariant is preserved (sketch the proof of the corresponding verification condition).

4. (25P) Take the following asynchronous composition of two processes operating on shared variables  $x, y, i, j$  where the first process cycles among program counters  $P_1 \rightarrow P_2 \rightarrow P_1 \rightarrow \dots$  and the second process among counters  $Q_1 \rightarrow Q_2 \rightarrow Q_3 \rightarrow Q_1 \rightarrow \dots$ .

```

initially x = y = i = 0, j = 1
loop          ||  loop
  P1: x = x+j;    ||    Q1: wait i > 0;
  P2: i = 1-i;    ||    Q2: y = y+i;
                     ||    Q3: j = 1-j;

```

a) (8P) Give a formal model of the system (using the interleaving assumption for asynchronous composition) as a *labeled* transition system with five transitions labeled  $P_1, P_2, Q_1, Q_2$ , and  $Q_3$ ; do not forget the definition of the state space.

b) (7P) Formalize in LTL the properties

- “ $i$  becomes greater than zero before  $y$  becomes greater than zero (which is eventually the case)”
- “if at any time  $i$  becomes greater than zero, then eventually also  $y$  will become greater than zero”.

c) (10P) Which of these properties are true without fairness requirements (if a property is not true, show a violating system run)? Which do become true, if we assume weak fairness for all transitions? Which do become true, if we assume strong fairness for all transitions? Explain your answers.