Computational Logic
Sample Exam Questions

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An exam has (100P) in total; the following questions amount to more than (100P).

1. (30P) Consider the following propositional formula F:

$$\neg(p \lor (q \land (r \lor s \Rightarrow p)))$$

   a) (6P) Give the NNF of F.
   b) (6P) Construct the truth table for F (it is not necessary to show the truth values of all subformulas).
   c) (6P) Determine from the truth table the DNF and the CNF of F.
   d) (12P) Derive the DNF and the CNF of F by logical equivalence transformations (show the main steps).

2. (24P) Consider the following propositional formula F:

$$((p \lor q) \land (\neg r \Rightarrow \neg p)) \Rightarrow (r \lor q)$$

   a) (6P) Prove the validity of F by a sequent calculus proof;
   b) (6P) Give the CNF of the negation of F.
   c) (6P) Prove the validity of F by a resolution proof.
   d) (6P) Prove the validity of F by applying the recursive DPLL algorithm (sketch the corresponding deduction tree).

3. (18P) Consider the following first-order formula F:

$$\neg(\forall x.\ p(x) \Rightarrow ((\forall y.\ r(x,y)) \lor (\exists y.\ q(x,y))))$$

   a) (6P) Give the NNF of F.

b) (6P) Give the PNF of F.

c) (6P) Give a formula F' in SNF that is equisatisfiable with F.

4. (15P) Consider the following first-order formula F:

$$\big(p(c) \wedge \forall x.p(x) \Rightarrow q(x, f(x))\big) \Rightarrow (\exists y.\ q(c, y))$$

Show the validity of F by applying the Gilmore algorithm.

5. (28P) Consider the following first-order formula F:

$$((\forall x.\ p(x) \Rightarrow q(x, f(x))) \wedge (\exists x.\ (\forall y.\ \neg q(x, y)))) \Rightarrow (\exists x.\ \neg p(x))$$

a) (8P) Prove the validity of F by a sequent calculus proof.

b) (8P) Prove the validity of F by the method of analytic tableaux (either the basic method or the free-variable method; indicate which variant you use).

c) (12P) Prove the validity of F by the resolution method.

6. (25P) Consider the following formula F in first-oder logic with equality:

$$\big(\forall x, y.\ e \circ x = x \wedge f(e) = e \wedge f(x \circ y) = f(y) \circ f(x)\big) \Rightarrow f(a \circ (b \circ e)) = f(b) \circ f(a)$$

a) (10P) Prove the validity of F by the method of analytic tableaux (either the basic method or the free-variable method; indicate which variant you use).

b) (15P) Prove the validity of F by paramodulation.

7. (10P) Consider the term rewriting system R induced by the following equations:

$$(x/y) * z = (x * z)/y \qquad (x/y) * y = x \qquad (x/x) = 1$$

a) (5P) Give the set of critical pairs of R.

b) (5P) Is R confluent? If not, add rewrite rules that make it confluent.

8. (10P) Consider the following formula F in theory LRA:

$$x \geq 1 \wedge 2x + 4y \leq 14 \wedge x - 2y \leq -1$$

Decide by the Fourier-Motzkin algorithm whether F is satisfiable (show the main steps). If the answer is positive, give a satisfying assignment for x and y.

9. (10P) Consider the following formula F in theory EUF:

$$a = b \wedge b = c \wedge g(f(a), b) = g(f(c), a) \Rightarrow f(a) = b$$

Decide by the congruence closure algorithm whether F is valid (show the partitions after each step of the algorithm).

10. (10P) Consider the following formula F in a combination of theories LRA and EUF:

$$a \leq b \wedge b \leq a \wedge g(a, b) = f(a) + f(b) \Rightarrow g(a, a) = 2 \cdot f(a)$$

Decide by the Nelson-Oppen Method the validity of F (show the main steps).