

Structured Global Programming for Communication Behaviour

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Abstract. This paper presents two different paradigms of description of communication behaviour, one focussing on global message flows and another on end-point behaviours, as formal calculi based on session types. The global calculus originates from Choreography Description Language, a web service description language developed by W3C WS-CDL working group. The end-point calculus is a typed π -calculus. The global calculus describes an interaction scenario from a vantage viewpoint; the endpoint calculus precisely identifies a local behaviour of each participant. After introducing the static and dynamic semantics of these two calculi, we explore a theory of endpoint projection which defines three principles for well-structured global description. The theory then defines a translation under the three principles which is sound and complete in the sense that all and only behaviours specified in the global description are realised as communications among end-point processes. Throughout the theory, underlying type structures play a fundamental role.

1 Introduction

Communication-Centred Programming. The explosive growth of Internet and World Wide Web in the last decades led to, in the form of de facto standards, an omnipresent naming scheme (URI/URL), omnipresent communication protocols (HTTP and TCP/IP) and an omnipresent message format (XML). These three elements offer the key infrastructural bases for application-level distributed programming. The software systems which make use of these and other common web standards for distributed communications are often called *web services*. Web services are an active area of infrastructural development, involving two major standardisation bodies, W3C and Oasis, and other private and public organizations.

One of the application domains which can naturally exploit the infrastructural basis of web services is the so-called business protocols. A business protocol is a series of structured and automated interactions among two or more business entities used for achieving their goals. Business protocols are inherently inter-domain, are often regulation-bound, and demand clear shared understanding about its meaning among multiple organisations with possibly conflicting interests. Numerous business protocols will be designed and implemented. Some business protocols such as industry standard will last long once specified; others would arise from temporary business needs and may undergo frequent updates. Because of its inherent inter-organizational nature, there is a strong demand for a common standard for specifying well-founded and correct business protocols.

Global Description of Interaction. One of the standardisation efforts for a language to specify business protocols is the Web Services Choreography Description Language (WS-CDL) [36], developed by W3C's WS-CDL Working Group since 2004 in collaboration with π -calculus experts as scientific advisors. WS-CDL is a specification language which directly describes global information flows and their structures, close to, for example, the standard notation for cryptographic protocols [23], UML sequence diagrams [24] and message sequence charts (MSC) [18]. Unlike these predecessors, in order to enable precise description and specification of complex business protocols, WS-CDL offers a fully expressive description language for channel based communication, equipped with standard control constructs (e.g. sequencing, conditionals and loops) and is conceived with potential for type-based and other formal validation. The underlying intuition behind *choreography* can be summarised as follows.

“Dancers dance following a global scenario without a single point of control.”

WS-CDL is a language for describing such a “global scenario” for business protocols. The description can then be executed by individual distributed processes without a single point of control.¹ Another significant feature of WS-CDL is its use of *sessions* for organizing communication behaviour: at the outset of each unit of a business protocol, a session is established between each pair of communication parties so that involved communications can be distinguished from any other instances of business protocols.

Endpoint Projection. A global description of communication behaviour arguably offers conceptual clarity not found in endpoint-based descriptions, partly because a global interaction flow *is* often the central objective a communication-based application is intended to realise. Real execution of the description, however, is always through communication among endpoints which (as the notion of choreography dictates) may as well involve no centralised control. Thus we ask:

How can we project a global description to endpoint processes so that their interactions precisely realise the original global description?

Such a projection may be called *endpoint projection (EPP)*, the term originating from WS-CDL WG.

What are criteria for a good EPP? We naturally desire an EPP to be *sound* and *complete*, in the sense that all and only globally described behaviour is realised as communications among endpoints. We may regard such an EPP as giving the semantics of a global description.

An appropriate notion of EPP leads to significant engineering usage of a global description:

1. (code generation) For a global description with full algorithmic details, we can create a (perhaps multi-language) *complete distributed application* by projecting it to each of its endpoints.

¹ An contrasting idea in web service is *orchestration* where one master component, “conductor”, directly controls activity of one or more slave components, which is useful in the intra-organisational applications.

2. (prototype generation) Projection can also be used for generating a *skeleton code* for each endpoint which only contains basic communication behaviour, to be elaborated to full code.
3. (conformance) A team of programmers initially agree on a shared global specification for interactions among endpoints: during/after programming, each programmer can check if her/his code conforms to the specification by conformance checking against projection. This scheme also applies to conformance of existing services/libraries to a given scenario.
4. (runtime monitoring, testing and debugging) At runtime, each endpoint can check if ongoing communications at his/her site conform to the global description by checking against its projection to that endpoint. The monitoring can also be used for debugging and testing existing code.
5. (property validation) Various static analyses/logical validation can be done for a global description so that they make sense for each endpoint through EPP.

Many of these ideas come from discussions in WS-CDL working group and are partly already realised in an open-source reference implementation of WS-CDL [25]. For example, runtime monitoring is a basic expected use of WS-CDL with relevance to regulatory concerns, especially for financial protocols. For all of these uses, EPP should be built on a clear, precise understanding of semantics of global and local descriptions, guaranteeing exact match between them.

This Work. The present paper introduces two typed calculi for interaction, one being a distillation of WS-CDL and another an applied version of the π -calculus, and develops a theory of endpoint projection. Our central contribution is the identification of natural descriptive principles for global descriptions which induce a type-preserving EPP that is sound and complete with respect to their operational semantics. There are three principles:

- *Connectedness*, a basic local causality principle.
- *Well-threadedness*, a stronger locality principle based on session types [9, 12, 13, 15, 33, 35].
- *Coherence*, a consistency principle for description of each participant in a global description.

These principles are stipulated incrementally on the basis of well-typedness. They not only enunciate natural disciplines for well-structured global description, but also offer gradually deeper analysis of global descriptions. The EPP has the following shape:

$$I \mapsto A[P] \mid B[Q] \mid C[R] \mid \dots$$

where I is a global description, A , B and C are *participants* to the protocol and P , Q and R are projections of I onto A , B and C respectively. When applied to well-structured interactions, the mapping thus defined satisfies the following three properties:

- *Type preservation*: the typing is preserved through EPP.
- *Soundness*: nothing but behaviours (reductions) in I are in the image of its EPP.
- *Completeness*: all behaviours in I are in the image of its EPP.

The EPP theory is intended as a theoretical basis of global description languages including, but not limited to, WS-CDL. The theory opens a conduit between global descriptions and accumulated studies on process calculi, allowing the exploitation of rich theories for engineering concerns. A version of EPP theory will be published as an associated document of WS-CDL 1.0, and will form a part of an open-source implementation of WS-CDL [25].

Related Works. As far as we know, this work is the first to present the typed calculus based on global description of communication behaviour, integrated with the theory of endpoint projection. Global methods for describing communication behaviour have been practiced in several different engineering scenes in addition to WS-CDL (for which this work is intended to serve as its theoretical underpinning). Representative examples include the standard notation for cryptographic protocols [23], message sequence charts (MSC) [18], and UML sequence diagrams [24]. These notations are intended to offer a useful aid at the design/specification stage, and do not offer full-fledged programming language, lacking in e.g. standard control structures and/or value passing. Petri-nets [34] may also be viewed as offering a global description, though again they are more useful as a specification/analytical tool.

DiCons (which stands for “Distributed Consensus”), which is independently conceived and predates WS-CDL, is a notation for global description and programming of Internet applications introduced and studied by Baeton and others [3]. DiCons chooses to use programming primitives close to user’s experience in the web, such as web server invocation, email, and web form filing, rather than general communication primitives. Its semantics is given by either MSCs or direct operational semantics. DiCons does not use session types or other channel-based typing. An analogue of the theory of endpoint projection has not been developed in the context of DiCons.

The present work shares with many recent works its direction towards well-structured communication-centred programming using types. Pict [28] is the programming language based on the π -calculus, with rich type disciplines including linear and polymorphic types (which come from the studies on types for the π -calculus discussed in the next paragraph). Polyphonic C# [4] uses a type discipline for safe and sophisticated object synchronisation. Compagnoni, Dezani, Gay, Vasconcelos and others have studied interplay of session type disciplines with different programming constructs and program properties [9, 12, 13, 15, 33, 35]. The EPP theory offers a passage through which these studies (all based on endpoint languages and calculi) can be reflected onto global descriptions, as we have demonstrated for session types in the present work. In the context of session types, the present work extends the session structure with multiple session names which is useful for having parallel communications inside a session.

Many theories of types for the π -calculus are studied. In addition to the study of session types mentioned above, these include input/output types [21, 27], linear types [14, 19], various kinds of behavioural types [2, 5, 6, 16, 17, 31, 32, 37] and combination of behavioural types and model checking for advanced behavioural analysis [29], to name a few. Among others, behavioural types offer an advanced analyses for such phenomena as deadlock freedom. We are currently studying how these advanced type-based validation techniques on the basis of the present simple session type discipline will

lead to effective validation techniques. Again these theories would become applicable to global descriptions through the link established by the EPP theory.

Gordon, Fournet, Bhargavan and Corin studied security-related aspects of web services in their series of works (whose origin lies in the security-enhanced pi-calculus called spi-calculus [1]). In their recent work [7], the authors have implemented part of WS-Security libraries using a dialect of ML, and have shown how annotated application-level usage of these security libraries in web services can be analysed with respect to their security properties by translation into the π -calculus [8]. The benefits of such a tool can be reflected onto the global descriptions through the theory of EPP, by applying the tool to projections.

Laneve and Padovani [20] give a model of orchestrations of web services using an extensions of π -calculus to join patterns. They propose a typing system for guaranteeing a notion of smoothness i.e. a constraint on input join patterns such that their subjects (channels) are co-located in order to avoid a classical global consensus problem during communication. Reflecting the centralised nature of orchestration (cf. footnote 1), neither a global calculus nor endpoint projection is considered. A bisimulation-based correspondence between choreography and orchestration in the context of web services has been studied in [10] by Busi and others, where a notion of state variables is used in the semantics of the orchestration model. They operationally relate choreographies to orchestration. Neither strong type systems nor disciplines for end-point projection are studied in their work.

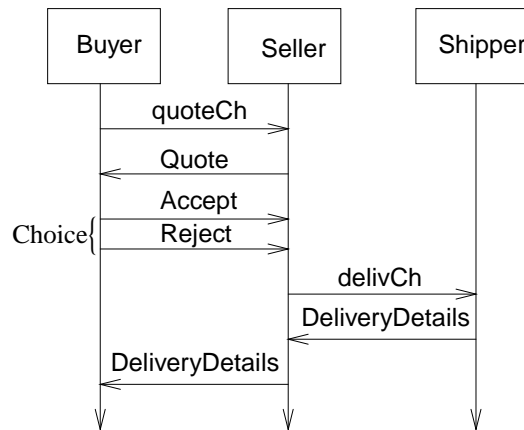
Outline. Section 2 previews the key technical ideas using concrete examples. Sections 3 and 4 outline the global and endpoint calculi, introducing their static and dynamic semantics. Section 5 develops the theory of endpoint projection. Section 6 summarises further results and applications of end-point projection. Section 7 concludes with further topics. The appendix presents a larger example illustrating how the endpoint projection concretely works. Many examples and the full technical developments of the theory are found in the full version [11].

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2 Preview of Key Technical Ideas

2.1 Buyer-Seller Protocol.

This section gives an outline of key technical ideas. Throughout we consider a simple business protocol from [30], which we call “Buyer-Seller Protocol”, and its variations. In the core protocol, the participants involved are a Buyer, a Seller and a Shipper. We first describe the protocol in the following sequence diagram.



In words, the protocol consists of the following actions.

- (1) Buyer asks Seller, through a specified channel, to offer a quote (denoted quote) for buying a specific good;
- (2) Seller replies with a quote;
- (3) Buyer then answers with either an accept or a reject.
- (4) *In the case of acceptance,*
 - (4-1) Seller sends the order to the Shipper;
 - (4-2) Shipper sends the delivery details back to the Seller.
 - (4-3) Seller forwards them to Buyer: the protocol terminates.
- (5) *In the case of rejection,* the protocol terminates.

Note the diagram is ambiguous on the branching Actions (4) and (5): the purpose of such diagrams is to offer an informal overview rather than precise specification. Nevertheless, protocols tend to be complex, with nondeterministic and conditional choices, loops, timeout and other elements. This motivates a need of a *syntactic means*, i.e. a language, for describing such protocols.

Some of the central elements of such a language (WS-CDL is one example) may be determined by observing that the whole intention of specifying such protocols is to *instantiate it repeatedly*, including its shared usage (e.g. the Buyer role can be assumed by any potential and possibly concurrent buyers). This consideration leads to the following two simple engineering principles.

Service Channel Principle (SCP): Invocation channels (e.g. a channel at which Buyer first communicates to Seller, similarly Seller to Shipper) can be shared and invoked repeatedly.

Session Principle (SP): A sequence of conversations belonging to a protocol should not be confused with other concurrent runs of this or other protocols by the participants: in other words, each such sequence should form one logical unit of a conversation, or a *session*.

(SCP) does not preclude a channel is only known to a closed number of participants. It corresponds to a replicated channel in the π -calculus, or, more accurately, a replicated

channel which is not prefixed by other input prefixes (such channels are called *uniformly receptive* in [31] and *server channels* in [5]). (**SP**) can have complex forms, but a most basic one is a dyadic one which allows simple and robust type abstraction with tractable type checking [12, 15, 35].² These two principles are central for the whole technical development in the paper.

2.2 A Language for Global Description of Communication.

The following presents the full description of Buyer-Seller Protocol in the *global calculus*, whose syntax and semantics we shall formally introduce later.

1. Buyer \rightarrow Seller : quoteCh(νs).
2. Seller \rightarrow Buyer : $s\langle\text{Quote}, 300, x\rangle$. {
3. {Buyer \rightarrow Seller : $s\langle\text{Accept}\rangle$.
4. Seller \rightarrow Shipper : delivCh(νt).
5. Shipper \rightarrow Seller : $t\langle\text{DelivDetails}, \nu_{\text{details}}, x_{\text{details}}\rangle$.
6. Seller \rightarrow Buyer : $s\langle\text{DelivDetails}, x_{\text{details}}, y_{\text{details}}\rangle$. **0** }
7. + }
8. {Buyer \rightarrow Seller : $s\langle\text{Reject}\rangle$. **0** }
9. }

Line 1 describes Action (1) in the previous informal description of the protocol. The quoteCh is a *service channel*, which may be considered as a publicly known URL for a specific service. The invocation marks the start of a session between the buyer and the seller: the ν -bound s is a *session name*, a fresh name that will be used for later communication in this session. Unlike standard process calculi, the syntax no longer describes input and output actions separately: the information exchange is directly described.

Line 2 describes Action (2) in the scenario, Seller's reply to Buyer. The session has already been started and now the two participants communicate using the session name s . In addition, three factors involved: Quote identifies the particular operation used in this communication (i.e. request of quote), 300 is the quote sent by Seller; x is a variable located at Buyer where the communicated value will be stored.

Lines 3/8 describe Action (3), where Buyer communicates its choice (Accept or Reject) to Seller through s . Two series of actions which follow these choices are combined by + in Line 7. If Accept is chosen, Seller sends Shipper the Buyer's details via the service channel delivCh of Shipper, creating a fresh session name t (Line 4). Then in Line 5, Shipper sends back the shipping details through t . Finally Seller forwards the details to Buyer in Line 6, where the protocol terminates. In Line 8, Buyer communicates Reject, in which case the protocol immediately terminates.

The code above offers a precise global description of the informal scenario above, drawing on (**SCP**) and (**SP**). Sessions offer logical grouping of threads of interactions,

² In implementations of web services, sessions are implemented using so-called *co-relation identities* (which may be considered as nonces in cryptographic protocols). This and the channel-based representation usually employed in the study of session types are logically equivalent, as discussed in [1].

where each thread starts with a procedure-call-like service invocation. This last feature can be seen more clearly in the following refinement of the code above.

```

1. Buyer → Seller : quoteCh( $\nu s$ ).
2. rec X. {
3. Seller → Buyer : s⟨Quote,  $q$ ,  $x$ ⟩.
4. if reasonable( $x$ )@Buyer then
5.   {Buyer → Seller : s⟨Accept⟩.
6.     Seller → Shipper : delivCh( $\nu t$ ).
7.     Shipper → Seller : t⟨DelivDetails,  $\nu_{\text{details}}$ ,  $x_{\text{details}}$ ⟩.
8.     Seller → Buyer : s⟨DelivDetails,  $x_{\text{details}}$ ,  $y_{\text{details}}$ ⟩. 0 }
9. else
10.  {Buyer → Seller : s⟨Reject⟩,  $q := q - 1$ @Seller. X } }
11. }
```

Above if Buyer chooses Reject, the protocol recurs to Line 3 after decrementing the quote. In Line 4, we assume a unary predicate `reasonable(x)` evaluated at Seller (“@” indicates a location, similarly in Line 10). Note the session notation makes it clear that all Quote-messages from Seller to Buyer in the recursion are done within a single session. Later in this preview we shall present another example where such session information plays a crucial role in tractable endpoint projection.

For comparison we present the endpoint counterpart of the first simple global code. The first is the endpoint code of Buyer.

$$\text{Buyer}[\overline{\text{QuoteCh}}(\nu s). s \triangleright \text{Quote}(x). \{ \\ \{ s \triangleleft \text{Accept}. s \triangleright \text{DeliveryDetails}(y_{\text{details}}). \mathbf{0} \} + \\ \{ s \triangleleft \text{Reject}. \mathbf{0} \} \}]$$

Above $\text{Buyer}[P]$ indicates a participant (a named agent) whose behaviour is given by P . The Seller’s code is given as:

$$\text{Seller}[\text{!QuoteCh}(s). \bar{s} \triangleleft \text{Quote}(300). \{ \\ \{ s \triangleright \text{Accept}. \\ \overline{\text{DeliveryCh}}(\nu t). t \triangleright \text{DeliveryDetails}(x_{\text{details}}). \\ \bar{s} \triangleleft \text{DeliveryDetails}(x_{\text{details}}). \mathbf{0} \} + \\ \{ \text{Reject}. \mathbf{0} \} \}]$$

The code of Shipper is similar. Observe endpoint descriptions clearly depict local communication behaviour. However they do not directly describe how interaction proceeds globally, which may often be the central concern of the designers and users of a communication-centred application. The two service channels (*QuoteCh* and *DeliverCh*) are replicated and ready to receive invocations, following (SCP).

As may be seen above, extraction of behaviour from a global description relies on session information. We illustrate this point further. Consider the following snippet of

global description, where a and b are used to indicate the lack of session information.

$$\begin{aligned} \text{Buyer} &\rightarrow \text{Seller} : a\langle \text{QuoteReq}, pname1, pname1 \rangle. \\ \text{Seller} &\rightarrow \text{Buyer} : b\langle \text{QuoteRes}, quote1, quote1 \rangle. \\ \text{Buyer} &\rightarrow \text{Seller} : a\langle \text{QuoteReq}, pname2, pname2 \rangle. \\ \text{Seller} &\rightarrow \text{Buyer} : b\langle \text{QuoteRes}, quote2, quote2 \rangle. I \end{aligned}$$

Here Buyer requests a quote twice: it may look that the behaviour of Seller is such that it allows a consecutive quote requests in one go. This ambiguity is resolved if we put a session information:

$$\begin{aligned} \text{Buyer} &\rightarrow \text{Seller} : ch(s)\langle \text{QuoteReq}, pname1, pname1 \rangle. \\ \text{Seller} &\rightarrow \text{Buyer} : s\langle \text{QuoteRes}, quote1, quote1 \rangle. \\ \text{Buyer} &\rightarrow \text{Seller} : ch(t)\langle \text{QuoteReq}, pname2, pname2 \rangle. \\ \text{Seller} &\rightarrow \text{Buyer} : t\langle \text{QuoteRes}, quote2, quote2 \rangle. I \end{aligned} \tag{1}$$

(Above we use a construct which combines a session initiation and an in-session communication. This is convenient for practice: our theoretical treatment in the present paper separates these two for clearer formal presentation, with no loss of generality via a simple encoding.) Using the session information, we infer:

$$!ch(s)[\langle \text{QuoteReq} \rangle(pname).\bar{s}\langle \text{QuoteRes} \rangle(quote).P]$$

Note the endpoint behaviour would have been quite different if we represent all request-replies as belonging to a single session.

2.3 Disciplines for Global Description.

Even if a global flow of interaction is the primary concern of an application designer, in implementation, a global scenario has to be realised by distributed end-points communicating with each other. Thus we need to bridge the world of global description to endpoint descriptions. Our ultimate aim is to have global descriptions such that their operational content, or endpoint realisation, is transparent from these descriptions.

Having such a bridge is non-trivial because a global calculus allows *description of communication behaviour that does not make sense at endpoints*. As the first such issue, let us consider the following code snippet for global description:

$$\begin{aligned} \text{Buyer} &\rightarrow \text{Seller} : ch_1(v s). \\ \text{Shipper} &\rightarrow \text{Depot} : ch_2(v t). \mathbf{0} \end{aligned}$$

Above Shipper is supposed to contact Depot only after Buyer performed a request to Seller. Implementing such a system demands Shipper is notified once the initial communication is performed, i.e. there is an implicit communication from Seller to Shipper:

$$\begin{aligned} \text{Buyer} &\rightarrow \text{Seller} : ch_1(v s). \\ \text{Seller} &\rightarrow \text{Shipper} : ch(v s'). \\ \text{Shipper} &\rightarrow \text{Depot} : ch_2(v t). \mathbf{0} \end{aligned}$$

With this insertion, the description is realisable purely through explicitly specified message exchanges. The criteria which says each participant acts only as a result of its local event (such as reception of a message) is called *connectedness*. We shall give its formal definition in Section 5.

Connectedness is an intuitive idea for well-structured global description. The next condition is more subtle. Consider the following (connected) interaction:

$$\begin{aligned} \text{Buyer} &\rightarrow \text{Seller} : ch_1(v\ s). \\ \text{Seller} &\rightarrow \text{Shipper} : ch_2(v\ t). \\ \text{Shipper} &\rightarrow \text{Buyer} : ch_3(v\ u). \\ \text{Buyer} &\rightarrow \text{Seller} : s\langle \text{op}, v, x \rangle. I \end{aligned}$$

Above we assume Buyer offers a service channel ch_3 which is useful for Shipper. We claim that this global code (regardless of ensuing interaction at I) is unrealisable at endpoints, at least under the natural type discipline and code organisation.

The first action tells that there is a thread in Buyer which invokes Seller. This thread becomes inactive in the second line. In the third line, a service at ch_3 in Buyer is invoked. In the final line, Buyer communicates to Seller via a session name s opened in the first action. So, at the endpoint, we should have the following two chunks of the code:

$$\overline{ch_1(v\ s)}. \bar{s} \triangleleft \text{op}(v).P \mid !ch_3(t).Q$$

The first chunk is for the initial invocation and ensuring reception of op in the same session, while the second is a service at ch_3 (by (SCP) this channel should be ready to receive invocations). Note that, by $\bar{s} \triangleleft \text{op}(v)$ belonging to a session s , this action cannot be located under ch_3 . On the other hand, the code of Seller should be:

$$!ch_1(s). \overline{ch_2(v\ t)}. t \triangleright \text{op}(x)$$

Finally, the code of Shipper becomes:

$$!ch_2(t). \overline{ch_3(v\ u)}.R$$

We can now have the three endpoint processes get engaged in communications: First, Buyer invokes ch_1 , then Seller invokes ch_2 of Shipper: up to here the interaction follows the original global scenario. However, at this point, the action $s \triangleright \text{op}(x)$ is free to react with its dual action $\bar{s} \triangleleft \text{op}(v)$, before Shipper invokes Seller's the other component, the service at ch_3 . Thus the sequencing in the global description can be violated.

The fundamental issue here is that the given global code assumes a false (unrealisable) dependency among actions: the last action belongs to a thread which started from the invocation of ch_1 , while the description says it should take place as the direct result of the third action at a distinct thread which has been opened by the invocation at ch_3 . If a global description is free from such false dependency, we say it is *well-threaded*. We shall see in Section 5 that checking well-threadedness is simple and mechanical.

Well-threadedness not only eliminates false dependency but also allows consistent extraction of threads (i.e. sequences of actions) from a given global code. These threads

become the constituents of endpoint processes in EPP. The final well-structuring principle is concerned with this composition. It is often necessary to *merge* threads to obtain the final endpoint behaviour of a single service.³ Consider the following parallel composition:

$$\begin{aligned} & \text{Buyer} \rightarrow \text{Seller} : ch(vs). \text{Seller} \rightarrow \text{Buyer} : s\langle op_1, e, x_1 \rangle. I_1 \mid \\ & \text{Buyer} \rightarrow \text{Seller} : ch(vt). \text{Seller} \rightarrow \text{Buyer} : t\langle op_2, e, x_2 \rangle. I_2 \end{aligned}$$

where $op_1 \neq op_2$. Here Buyer invokes Seller's service at ch twice in parallel. Now consider constructing the code for this service at channel ch : then we need to merge these two threads into one endpoint behaviour. But the global description is contradictory, since in one invocation the service reacts with op_1 , while in another the service reacts with op_2 . The description is not self-consistent.

A central issue is that, in a global description, the descriptions of a single endpoint behaviour (especially a service at a service channel) can be *scattered in different portions of the code*. Thus, without these scattered descriptions being consistent with each other, we cannot merge them into a single behaviour. We call such mergeability, *coherence*: coherence is not simply about identity of the behaviour, as in the above case, since distinct input branches may be described in different portions of a global code. The details are given in Section 5. Coherence can again be checked mechanically.

With coherence as the final well-structuredness condition, we can now project a global code to endpoint behaviours that precisely realise the original global scenario.

3 The Gobal Calculus

3.1 Syntax

The syntax of the global calculus is given by the following BNF. I, I', \dots denote *terms* of the calculus, also called *interactions*. Below ch, ch', \dots range over *service channels*, intuitively denoting the shared channels of (web) services; s, t, \dots range over *session names*; \tilde{s} indicates a vector of session names; A, B, C, \dots range over *participants*; x, y, z, \dots over local variables in each participant; X, X', \dots over *term variables*; and e, e', \dots over arithmetic and other first-order expressions.

$$\begin{aligned} I ::= & A \rightarrow B : ch(v\tilde{s}). I && \text{(init)} \\ & | A \rightarrow B : s\langle op, e, y \rangle. I && \text{(comm)} \\ & | x@A := e. I && \text{(assign)} \\ & | I_1 \mid I_2 && \text{(par)} \\ & | \text{if } e@A \text{ then } I_1 \text{ else } I_2 && \text{(ifthenelse)} \\ & | I_1 + I_2 && \text{(sum)} \\ & | (vs) I && \text{(new)} \\ & | X && \text{(recvar)} \\ & | \mu X. I && \text{(rec)} \\ & | \mathbf{0} && \text{(inaction)} \end{aligned}$$

³ This merging already takes place in the extraction of code in (1) above, though in a trivial way.

Table 1 Reduction Relation for the Global Calculus

$(G-I) \frac{-}{(\sigma, A \rightarrow B : ch(\nu \bar{s}), I) \rightarrow (\sigma, (\nu \bar{s}) I)}$	$(G-C) \frac{\sigma' = \sigma[x@B \mapsto v] \quad \sigma \vdash e@A \Downarrow v}{(\sigma, A \rightarrow B : s(\text{op}, e, x), I) \rightarrow (\sigma', I)}$
$(G-A) \frac{\sigma' = \sigma[x@A \mapsto v] \quad \sigma \vdash e@A \Downarrow v}{(\sigma, x@A := e, I) \rightarrow (\sigma', I)}$	$(G-S) \frac{i = 1, 2}{(\sigma, I_1 + I_2) \rightarrow (\sigma', I_i)}$
$(G-I T) \frac{\sigma \vdash e@A \Downarrow \text{tt}}{(\sigma, \text{if } e@A \text{ then } I_1 \text{ else } I_2) \rightarrow (\sigma, I_1)}$	$(G-I F) \frac{\sigma \vdash e@A \Downarrow \text{ff}}{(\sigma, \text{if } e@A \text{ then } I_1 \text{ else } I_2) \rightarrow (\sigma, I_2)}$
$(G-P) \frac{(\sigma, I_1) \rightarrow (\sigma', I'_1)}{(\sigma, I_1 I_2) \rightarrow (\sigma', I'_1 I_2)}$	$(G-R) \frac{(\sigma, I) \rightarrow (\sigma', I')}{(\sigma, (\nu \bar{s}) I) \rightarrow (\sigma', (\nu \bar{s}) I')}$
$(G-R) \frac{(\sigma, I[\mu X.I/X]) \rightarrow (\sigma', I')}{(\sigma, \mu X.I) \rightarrow (\sigma', I')}$	$(G-S) \frac{I \equiv I'' \quad (\sigma, I) \rightarrow (\sigma, I') \quad I' \equiv I'''}{(\sigma, I'') \rightarrow (\sigma', I''')}$

The term (init) denotes a session initiation by A to B on service channel ch with fresh session channels \bar{s} and continuation I . The interaction (com) is the in-session communication over a session channel s . Note that y is free and does not bind I . The operators $|$ and $+$ denote respectively parallel and choice operators. $(\nu \bar{s}) I$ is the π -calculus-like name restriction, binding s in I . (ifthenelse) and (assign) are the standard conditional and assignment ($e@A$ indicates e is located at A). $\mu X. I$ is recursion, where the variable X is bound in I . $\mathbf{0}$ denotes termination. The free and bound session channels and term variables are defined in the usual way.

The presented syntax is intended as the minimum one for presenting examples and for the EPP theory. Section 6 discusses natural additional syntactic constructs.

3.2 Dynamics

The dynamics of the global calculus is given by reduction relation close to that of imperative languages. A *state* σ assigns a value to the variables located at each participant. We shall write $\sigma@A$ to denote the portion of σ local to A , and $\sigma[y@A \mapsto v]$ to denote a new state which is identical with σ except that $\sigma'@A(y)$ is equal to v . The reduction is the binary relation \rightarrow generated by the rules in Table 1. “ $(\sigma, I) \rightarrow (\sigma', I')$ ” says that I in the state σ performs one-step computation and becomes I' with the new state σ' .

Rule (I) is for session initiation: after A initiates a session with B on service channel ch , A and B share \bar{s} locally (indicated by binding $(\nu \bar{s})$), and the next I is unfolded. The initiation channel ch will play an important role for typing and the end-point projection later. (C) is a key rule: the expression e is evaluated into v in the A -portion of the state σ and then assigned to the variable x located at B resulting in new the state $\sigma[x@B \mapsto v]$. Note that the same variable (say x) can be located at different participants, so that $\sigma@A(x)$ and $\sigma@B(x)$ are distinct. Similarly to the session initiation, the session channel s is attached. The rule (S) makes use of structural congruence. The structural congruence relation is the least congruence relation \equiv on I

Table 2 Typing Rules for Global Calculus

$$\begin{array}{c}
\text{(G-TI)} \quad \frac{\Gamma, ch@B:(\tilde{s})\alpha \vdash I \triangleright \Delta \cdot \tilde{s}[B, A]:\alpha}{\Gamma, ch@B:(\tilde{s})\alpha \vdash A \rightarrow B : ch(\nu \tilde{s}). I \triangleright \Delta} \\
\text{(G-TC)} \quad \frac{\Gamma \vdash I \triangleright \Delta \cdot \tilde{s}[A, B]:\alpha_j \quad \Gamma \vdash e@A:\theta_j \quad \Gamma \vdash x@B:\theta_j \quad s \in \{\tilde{s}\} \quad j \in J}{\Gamma \vdash A \rightarrow B : s\langle \text{op}_j, e, x \rangle. I \triangleright \Delta \cdot \tilde{s}[A, B]:s \blacktriangleleft \Sigma_{i \in J} \text{op}_i(\theta_i). \alpha_i} \\
\text{(G-TC I)} \quad \frac{\Gamma \vdash I \triangleright \Delta \cdot \tilde{s}[B, A]:\alpha_j \quad \Gamma \vdash e@A:\theta_j \quad \Gamma \vdash x@B:\theta_j \quad s \in \{\tilde{s}\} \quad j \in J}{\Gamma \vdash A \rightarrow B : s\langle \text{op}_j, e, x \rangle. I \triangleright \Delta \cdot \tilde{s}[B, A]:s \blacktriangleright \Sigma_{i \in J} \text{op}_i(\theta_i). \alpha_i} \\
\text{(G-TA)} \quad \frac{\Gamma \vdash x@A:\theta \quad \Gamma \vdash e@A:\theta \quad \Gamma \vdash I \triangleright \Delta}{\Gamma \vdash x := e@A. I \triangleright \Delta} \\
\text{(G-TS)} \quad \frac{\Gamma \vdash I_1 \triangleright \Delta \quad \Gamma \vdash I_2 \triangleright \Delta}{\Gamma \vdash I_1 + I_2 \triangleright \Delta} \quad \text{(G-TI)} \quad \frac{\Gamma \vdash e@A:\text{bool} \quad \Gamma \vdash I_1 \triangleright \Delta \quad \Gamma \vdash I_2 \triangleright \Delta}{\Gamma \vdash \text{if } e@A \text{ then } I_1 \text{ else } I_2 \triangleright \Delta} \\
\text{(G-TP)} \quad \frac{\Gamma \vdash I_1 \triangleright \Delta_1 \quad \Gamma \vdash I_2 \triangleright \Delta_2}{\Gamma \vdash I_1 | I_2 \triangleright \Delta_1 \bullet \Delta_2} \quad \text{(G-TR 1)} \quad \frac{\Gamma \vdash I \triangleright \Delta, \tilde{s}_1 \tilde{s}_2[A, B]:\alpha}{\Gamma \vdash (\nu s) I \triangleright \Delta, \tilde{s}_1 \tilde{s}_2:\perp} \\
\text{(G-TR 2)} \quad \frac{\Gamma \vdash I \triangleright \Delta, \tilde{s}_1 \tilde{s}_2:\perp}{\Gamma \vdash (\nu s) I \triangleright \Delta, \tilde{s}_1 \tilde{s}_2:\perp} \quad \text{(G-TR 3)} \quad \frac{\Gamma \vdash I \triangleright \Delta, \varepsilon:\perp}{\Gamma \vdash (\nu s) I \triangleright \Delta} \\
\text{(G-TR)} \quad \frac{\Gamma \cdot X:\Delta \vdash I \triangleright \Delta}{\Gamma \vdash \mu X. I \triangleright \Delta} \quad \text{(G-TV)} \quad \frac{\Gamma, X:\Delta \text{ well-formed}}{\Gamma, X:\Delta \vdash X \triangleright \Delta} \\
\text{(G-TZ)} \quad \frac{\Gamma \text{ well-formed} \quad \forall i \neq j. \{\tilde{s}_i\} \cap \{\tilde{s}_j\} = \emptyset}{\Gamma \vdash \mathbf{0} \triangleright \bigcup_i \tilde{s}_i[A_i, B_i] \text{end}}
\end{array}$$

such that $|$ and $+$ are commutative monoids and such that it satisfies alpha-conversion and the rule $(\nu s) I_1 | I_2 \equiv (\nu s) (I_1 | I_2)$ for $s \notin \text{fn}(I_2)$.

Consider, for instance, the interaction

$$\begin{array}{l}
\text{Buyer} \rightarrow \text{Seller} : \text{QuoteCh}(\nu s). \\
\text{Seller} \rightarrow \text{Buyer} : s\langle \text{Quote}, 300, x \rangle. I'
\end{array}$$

and let us evaluate it in the state σ . By applying rule (I), we get the pair $(\sigma, (\nu s) \text{Seller} \rightarrow \text{Buyer} : s\langle \text{Quote}, 300, x \rangle. I')$. Now, by applying rules (R) and (C) together in the state σ we get the pair $(\sigma[x@\text{Buyer} \mapsto 300], (\nu s) I')$.

3.3 Typing

We use a generalisation of session types [15] as the type discipline for the global calculus. The grammar of types follows.

$$\begin{array}{l}
\alpha ::= s \blacktriangleright \Sigma_i \text{op}_i(\theta_i). \alpha_i \quad | \quad s \blacktriangleleft \Sigma_i \text{op}_i(\theta_i). \alpha_i \\
\quad | \quad \alpha | \alpha \quad | \quad \text{end} \quad | \quad \mu \mathbf{t}. \alpha \quad | \quad \mathbf{t}
\end{array}$$

where θ, θ', \dots range over *value types*. α, α', \dots are *session types*. $s \blacktriangleright \Sigma_i \text{op}_i(\theta_i)$. α_i is a *branching input type* at session channel s , indicating possibilities for receiving any of the operators from op_i (which are pairwise distinct) with a value of type θ_i ; $s \blacktriangleleft \Sigma_i \text{op}_i(\theta_i)$. α_i , a *branching output type* at s , is the exact dual of the above. The type $\alpha_1 | \alpha_2$ is a *parallel composition of α_1 and α_2* , abstracting parallel composition of two sessions. We take $|$ to be commutative and associative, with end , the *inaction type* indicating session termination, being the identity. We demand session channels in α_1 and those in α_2 to be disjoint: this will guarantee a linear use of session channels. \mathbf{t} is a *type variable*, while $\mu \mathbf{t}.\alpha$ is a *recursive type*, where $\mu \mathbf{t}$ binds free occurrences of \mathbf{t} in α . In recursive types, we assume each recursion is guarded, i.e., in $\mu \mathbf{t}.\alpha$, α is an n -ary parallel composition of input/output types. Recursive types are regarded as regular trees in the standard way [26].

Note that session channels occur free in session types: this is necessary to allow multiple session channels to appear in a single session in parallel. Thus, we can faithfully capture the behaviour of web services where it is possible to exchange different data simultaneously, leading to a generalisation of session types in the literature. Let us show a simple example:

$$s \blacktriangleleft \text{Quote}(\text{int}). \text{end} \mid s' \blacktriangleleft \text{Extra}(\text{String}). \text{end}$$

Here a participant is sending a quote (integer) at s and extra information about the product at s' in a single session: without using distinct session channels, two communications can get confused.

The duality for session types plays the key role to guarantee dyadic interaction [15]. The *co-type*, or *dual*, of α , written $\bar{\alpha}$, is given as follows.

$$\begin{aligned} \overline{s \blacktriangleleft \Sigma_i \text{op}_i(\theta_i). \alpha_i} &= s \blacktriangleright \Sigma_i \text{op}_i(\theta_i). \bar{\alpha}_i \\ \overline{s \blacktriangleright \Sigma_i \text{op}_i(\theta_i). \alpha_i} &= s \blacktriangleleft \Sigma_i \text{op}_i(\theta_i). \bar{\alpha}_i \\ \overline{\mu \mathbf{t}.\alpha} &= \mu \mathbf{t}.\bar{\alpha} \quad \overline{\mathbf{t}} = \mathbf{t} \quad \overline{\text{end}} = \text{end} \end{aligned}$$

For instance, the co-type of $s \blacktriangleright \text{Quote}(\text{int}). \text{end}$ is $s \blacktriangleleft \text{Quote}(\text{int}). \text{end}$, exchanging input and output.

Each time a session is initiated via a service channel, session channels are freshly generated. Thus, the interface of a service should indicate a vector of session names to be exchanged, in addition to how they are used. This is represented by a *service type*, in which concrete instances of session names in a session type are abstracted, written: $(\tilde{s})\alpha$ where \tilde{s} is a vector of pairwise distinct session channels covering all session channels in α , and α does not contain free type variables. (\tilde{s}) binds occurrences of session channels in \tilde{s} in α , which induces the standard alpha-equality.

A *typing judgement* has the following form:

$$\Gamma \vdash I \triangleright \Delta$$

where Γ is *service typing* and Δ *session typing*. Δ maps session channels to their locations and session types and Γ located service channels and recursive variables to service types and session typing, respectively. The grammar of service/session typings follow.

Below in $\tilde{s}[A, B]$ we assume $A \neq B$.

$$\begin{aligned}\Gamma &::= \emptyset \mid \Gamma, ch@A:(\tilde{s})\alpha \mid \Gamma, x@A:\theta \mid \Gamma, X:\Delta \\ \Delta &::= \emptyset \mid \Delta, \tilde{s}[A, B]:\alpha \mid \Delta, \tilde{s}:\perp\end{aligned}$$

In a service typing, three forms of assignments are used: $ch@A:(\tilde{s})\alpha$ says that a service channel ch is located at A and offers a service interface represented by a service type $(\tilde{s})\alpha$; $x@A:\theta$ says that a variable x located at A may store values of type θ ; finally, $X:\Delta$ is for recursion i.e. when the interaction recurs to X , the behaviour will own a session typing Δ .

A session typing uses the primary form of assignment $\tilde{s}[A, B]:\alpha$ which says that a vector of session channels \tilde{s} , all belonging to a same session which is between A and B , has the session type α when seen from the viewpoint of A . We write $\Gamma_1, \Gamma_2 (\Delta_1, \Delta_2)$ if there is no overlap between the free variables/names in $\Gamma_1 (\Delta_1)$ and $\Gamma_2 (\Delta_2)$. The notation $\text{fsc}(\Delta)$ denotes the set of free service/session channels in Δ .

The typing rules are given in Table 2. Rule (G-TC₁) states that I should contain a session type α_j between A and B such that its session channels contain s . The communicated value e is typed in the source (A) while the variable x is typed in the target (B), with the same type θ_j . In the conclusion, a branching type should include the operator op_j whose value type is θ_j . In (G-TC₂), the session type in focus is given with the direction from A to B , i.e. it abstracts the structure of the interaction in this session from the viewpoint of A . While this is consistent, we may also regard it from the receiver viewpoint (B). Thus we have the symmetric variant (G-TC₁ I).

Rule (G-TP₁) uses the linearity condition found in [15]. The operator \bullet is well-defined whenever the linearity condition is satisfied and is such that $\tilde{s}[A, B]:\alpha \in \Delta_1 \bullet \Delta_2$ iff either

1. $\tilde{s}[A, B]:\alpha_1 \in \Delta_1, \tilde{s}[A, B]:\alpha_2 \in \Delta_2$ and $\alpha = \alpha_1 \mid \alpha_2$;
2. $\tilde{s}[A, B]:\alpha \in \Delta_1$ and $\{\tilde{s}\} \cap \text{fsc}(\Delta_2) = \emptyset$, or its symmetric case;

Note different session channels can be used in parallel, while service channels can be shared by multiple threads of interactions.

Rule (G-TI₁) types session initiation. Since \tilde{s} is to be abstracted as session channels belonging to a single session, we demand that there is a $ch@B:(\tilde{s})\alpha$ in the typing environment. Since \tilde{s} is directed from B to A , α designates a session type seen from B 's viewpoint resulting in $s[B, A]:\alpha$ where both A and B need be mentioned since a session is always between two parties. Note that $ch@B:(\tilde{s})\alpha$ is also assumed in the premise since ch may have already been used elsewhere (as a service channel can be shared).

In (G-TR₁), hiding of session names is introduced after the session initiation so that they can no longer be abstracted by (G-TI₁). α is no longer necessary, so we replace it with \perp . Rule (G-TR₂) is used for removing unnecessary hidden session names one by one: when \tilde{s} is empty, we take it off with (G-TR₃).

In Rule (G-TZ₁) we demand each session type used in the conclusion is a distinct vector of session channels and Γ is well-formed. A service type Γ is *well-formed* whenever $ch@A_i:(\tilde{s}_i)\alpha_i \in \Gamma$ ($i = 1, 2$) implies $A_1 = A_2$ and $(\tilde{s}_1)\alpha_1 = (\tilde{s}_2)\alpha_2$. Moreover, $x@A_i:\theta_i, X:\Delta_i \in \Gamma$ implies $\theta_1 = \theta_2, A_1 = A_2$ and $\Delta_1 = \Delta_2$. Similarly, a session typing Δ

is *well-formed* when for all $\tilde{s}_1[A_1, B_1]\alpha_1$ and $\tilde{s}_2[A_2, B_2]\alpha_2$ in Δ such that $\{\tilde{s}_1\} \cap \{\tilde{s}_2\} \neq \emptyset$ we have $\tilde{s}_1 = \tilde{s}_2$, $A_1 = A_2$, $B_1 = B_2$ and $\alpha_1 = \alpha_2$.

Proposition 1. $\Gamma \vdash I \triangleright \Delta$ implies Γ and Δ are well-formed.

As a simple example, we type the Buyer-Seller interaction I . Service channel QuoteCh is assigned with service type

$$\begin{aligned} (s) s \blacktriangleleft \text{Quote}(\text{integer}). (\\ s \blacktriangleright \text{QuoteAccept}(\text{null}). s \blacktriangleleft \text{DeliveryDetails}(\text{null}). \text{end} + \\ s \blacktriangleright \text{QuoteReject}(\text{string}). \text{end}) \end{aligned}$$

Instead, service channel DeliveryCh has service type

$$(t) t \blacktriangleleft \text{DeliveryDetails}(\text{string}). \text{end}$$

Denoting two types by $(s)\alpha_1$ and $(t)\alpha_2$ respectively, we can prove $\text{QuoteCh}:(s)\alpha_1, \text{DeliveryCh}:(t)\alpha_2 \vdash I \triangleright \emptyset$.

The type discipline has also a minimal typing. To formulate minimality, we use the *inclusion ordering* \leq , defined based on simulation as in [13] with the key justifying rules being:

$$\frac{J \subset J' \quad \forall i \in J. \alpha_i \leq \alpha'_i}{s \blacktriangleright \sum_{i \in J} \text{op}_i(\theta_i). \alpha_i \leq s \blacktriangleright \sum_{i \in J'} \text{op}_i(\theta_i). \alpha'_i}$$

$$\frac{J \subset J' \quad \forall i \in J. \alpha_i \leq \alpha'_i}{s \blacktriangleleft \sum_{i \in J} \text{op}_i(\theta_i). \alpha_i \leq s \blacktriangleleft \sum_{i \in J'} \text{op}_i(\theta_i). \alpha'_i}$$

The relation \leq is extended pointwise to session typings and service typings. In brief, $\alpha \leq \alpha'$ indicates α is the result of cutting off some branches from α' at zero or more points. We now observe:

Proposition 2. Let $\Gamma \vdash I \triangleright \emptyset$ for some Γ . Then there exists Γ_0 such that $\Gamma_0 \vdash I$ and whenever $\Gamma' \vdash I \triangleright \emptyset$ we have $\Gamma_0 \leq \Gamma'$.

Theorem 3 (Subject Reduction). Assume $\Gamma \vdash \sigma$. Then $\Gamma \vdash I \triangleright \Delta$ and $(\sigma, I) \rightarrow (\sigma', I')$ imply $\Gamma \vdash \sigma'$ and $\Gamma \vdash I \triangleright \Delta'$ for some Δ' such that $\text{fsc}(\Delta') \subset \text{fsc}(\Delta)$.

4 The End-Point Calculus

4.1 Syntax

The end-point calculus is an applied version of the π -calculus [22]. The main syntactic terms are *processes* (P, Q, \dots) and *networks* (M, N, \dots) and are defined by the following grammar.

$$\begin{aligned} P ::= & !ch(\tilde{s}). P \mid \overline{ch}(\nu\tilde{s}). P \mid s \triangleright \sum_i \text{op}_i(\tilde{y}_i). P_i \\ & \mid \overline{s} \triangleleft \text{op}(\tilde{e}). P \mid x := e. P \mid \text{if } e \text{ then } P_1 \text{ else } P_2 \\ & \mid P \oplus P \mid P \mid Q \mid (\nu s) P \mid X \mid \mu X. P \mid \mathbf{0} \\ N ::= & A[P]_\sigma \mid N \mid N \mid (\nu s) N \mid \epsilon \end{aligned}$$

Table 3 Semantics of the End-Point Calculus

$$\begin{array}{c}
 \text{(EP-I)} \quad \frac{}{A[!ch(\tilde{s}). P | P']_{\sigma} | B[\overline{ch}(\nu\tilde{s}). Q | Q']_{\sigma'} \rightarrow (\nu\tilde{s}) (A[!ch(\tilde{s}). P | P']_{\sigma} | B[Q | Q']_{\sigma'})} \\
 \\
 \text{(EP-C)} \quad \frac{\sigma \vdash e \Downarrow v}{A[s \triangleright \Sigma_i \text{op}_i(x_i). P_i | P']_{\sigma} | B[\tilde{s} \triangleleft \text{op}_i(e) Q | Q']_{\sigma'} \rightarrow A[P_j | P']_{\sigma[x \mapsto v_j]} | B[Q | Q']_{\sigma'}} \\
 \\
 \text{(EP-A)} \quad \frac{\sigma \vdash e \Downarrow v}{A[x := e. P | P']_{\sigma} \rightarrow A[P | P']_{\sigma[x \mapsto v]}}
 \end{array}$$

The first two productions for processes describe terms meant for session initiation and the following two are for communication. This is in the style of [15], except \tilde{y}_i in the second construct (branching input) do *not* induce binders. $x := e. P$ assigns a value v to x in its store and then continues as P . The rest is all standard. Networks are parallel composition of participants. The latter are represented by the term $A[P]_{\sigma}$ which indicates a participant A whose behaviour is given by P and whose local state is σ .

4.2 Reduction

The reduction semantics for the end-point calculus follows the π -calculus. We report the full definition in Appendix B, but list the three key rules in Table 3.

(EP-I) defines the session initiation: two participant A and B will synchronize starting a session whenever they are executing a service offer $!ch(\tilde{s}). P$ and a request of service $\overline{ch}(\nu\tilde{s}). Q$ respectively. The synchronisation will result into sharing fresh session names \tilde{s} local to A and B . These session names are then used in (EP-C) for communication. In (EP-C), we use assignment to local variables rather than value passing for correspondence with the global calculus. (EP-A) updates the store associated in each participant.

4.3 Session Types for the End-Point Calculus

As mentioned above, the aim of the end-point calculus is to give a model on which we can project the global calculus. For this reason we need to define session types [15] as well. We use the same set of session and service types as the global calculus.

In the end-point calculus, we have the two type judgements

$$\Gamma \vdash_A P \triangleright \Delta \qquad \Gamma \vdash M \triangleright \Delta$$

respectively for processes and networks. Γ (service typing) and Δ (session typing) above are given by the following grammar.

$$\begin{array}{l}
 \Gamma ::= \emptyset \mid \Gamma, ch@A : (\tilde{s})\alpha \mid \Gamma, \overline{ch}@A : (\tilde{s})\alpha \\
 \quad \mid \Gamma, x@A : \theta \mid \Gamma, X : \Delta \\
 \Delta ::= \emptyset \mid \Delta, \tilde{s}@A : \alpha \mid \Delta, \tilde{s} : \perp
 \end{array}$$

Table 4 Session Types for Processes in the End-Point Calculus

$$\begin{array}{l}
 \text{(EP-TB)} \quad \frac{K \subseteq J \quad s \in \tilde{s} \quad \Gamma \vdash x_j : \theta_j \quad \Gamma \vdash_A P_j \triangleright \Delta \cdot \tilde{s}@A : \alpha_j}{\Gamma \vdash s \triangleright \sum_{i \in J} \text{op}_i(x_i). P_i \triangleright \Delta \cdot \tilde{s}@A : s \blacktriangleright \sum_{i \in K} \text{op}_i(\theta_i). \alpha_i} \\
 \\
 \text{(EP-TS)} \quad \frac{j \in J \subseteq K \quad \Gamma \vdash e : \theta_j \quad \Gamma \vdash_A P \triangleright \Delta \cdot \tilde{s}@A : \alpha_j}{\Gamma \vdash_A \bar{s} \triangleleft \text{op}_j(e). P \triangleright \Delta \cdot \tilde{s}@A : s \blacktriangleleft \sum_{i \in K} \text{op}_i(\theta_i). \alpha_i} \\
 \\
 \text{(EP-TS)} \quad \frac{\Gamma \vdash_A P \triangleright \tilde{s}@A : \alpha}{\Gamma, \text{ch}@A : (\tilde{s})\alpha \vdash_A !\text{ch}(\tilde{s}). P \triangleright \emptyset} \\
 \\
 \text{(EP-TR)} \quad \frac{\Gamma, \overline{\text{ch}}@B : (\tilde{s})\alpha \vdash_A P \triangleright \Delta \cdot \tilde{s}@A : \alpha}{\Gamma, \overline{\text{ch}}@B : (\tilde{s})\alpha \vdash_A \overline{\text{ch}}(\nu\tilde{s}). P \triangleright \Delta} \\
 \\
 \text{(EP-TP)} \quad \frac{\Gamma_i \vdash_A P_i \triangleright \Delta_i \quad \Gamma_1 \asymp \Gamma_2 \quad \Delta_1 \asymp \Delta_2}{\Gamma \vdash_A P_1 \mid Q_2 \triangleright \Delta_1 \odot \Delta_2}
 \end{array}$$

As before, we stipulate that, whenever we write e.g. Γ_1, Γ_2 , there are *no* free channel-s/variables shared between two typings. The selected rules for the typing processes are given in Table 4.

The rule (EP-TB) for input in-session communication involves branching with distinct operators: the typing can have less branches than the real process, so that the process is prepared to receive any operator specified in the type. Rule (EP-TS) is its dual: the typing can have more branches than the real process, so that the process invokes with operators at most those specified in types. Combining (EP-TB) and (EP-TS), an output never tries to invoke a non-existent option in its matching input.

Rule (EP-TS) is for typing the inputting side of initialisation. Note we do not allow those session channels other than the target of initialisation to be present as the session typing in the premise: this prevents *free* session channels to be under the replicated input, guaranteeing their linear usage. The typing in the conclusion means (by our convention) that *ch* or $\overline{\text{ch}}$ does not occur in Γ . The outputting side of initialisation (rule (EP-TR)) is analogous, except that the linearity constraint needs not be specified. We assume that A and B are not identical. The fact we allow $\overline{\text{ch}}@B : (\tilde{s})\alpha$ to occur in the premise means an invocation to a service can be done as many times as needed (as far as it is type correct). (EP-TP) uses the operators \asymp and \odot : $\Delta_1 \asymp \Delta_2$ means two channels of the same domain have a dual type each other [15]; and the result of the composition of the dual types become \perp , which denotes the same channel cannot be composable further. This operation guarantees a linear use of session channels. The full definition can be found in Appendix B.

As an example, we type the end-point process of the seller, seen in Section 2. If we consider the service types $(s)\alpha_1$ and $(t)\alpha_2$ in the previous example in Section 3.3, we have

$$\text{QuoteCh} : (s)\alpha_1, \overline{\text{DeliveryCh}} : (t)\bar{\alpha}_2 \vdash \text{Seller[Protocol]}_\sigma \triangleright \emptyset$$

Note that, in the end-point types the service channel $\overline{\text{DeliveryCh}}$ is overlined: this is because the channel is located at the shipper's. This is not the case in the global calculus as we only have a global view of channels. With well-formedness similar to the global calculus, we have:

Proposition 4. $\Gamma \vdash M \triangleright \Delta$ implies Γ and Δ are well-formed.

For the end-point calculus, we consider a subtyping relation on session types following [13].⁴ This relation plays a basic role in our subsequent technical development. The subtyping is written $\alpha \leq \beta$. Intuitively, $\alpha_1 \leq \alpha_2$ indicates that α_1 is more gentle, or dually α_2 is less constrained, in behaviour. The subtyping relation is given based on simulation following [13], whose key justifying rules are:

$$\frac{J \supset J' \quad \alpha_i \leq \beta_i}{s \blacktriangleright \Sigma_{i \in J} \text{op}_i(\theta_i). \alpha_i \leq s \blacktriangleright \Sigma_{i \in J'} \text{op}_i(\theta_i). \beta_i}$$

which says that if the initial input offers more options, and if subsequent behaviours are more gentle, then it is more gentle.

$$\frac{J \subset J' \quad \alpha_i \leq \beta_i}{s \blacktriangleleft \Sigma_{i \in J} \text{op}_i(\theta_i). \alpha_i \leq s \blacktriangleleft \Sigma_{i \in J'} \text{op}_i(\theta_i). \beta_i}$$

which says that if the initial output has less emissions and if subsequent behaviours are more gentle then it is more gentle. Note this relation is different from the inclusion ordering \leq in §3.3.

The following result says that we can always find a representative typing for a given process, and, moreover, we can do so effectively. Such a type is minimum among all assignable typings w.r.t. the subtyping relation, so that we call it the *minimal typing* of a given term. Below and henceforth we write $\Gamma \vdash M$ for $\Gamma \vdash M \triangleright \emptyset$, similarly for $\Gamma \vdash_A P$.

Definition 5 (minimal typing). Let $\Gamma_0 \vdash M$. We call Γ_0 the minimal service typing of M whenever for all Γ such that $\Gamma \vdash M$ we have $\Gamma_0 \leq \Gamma$, where \leq is taken pointwise at each channels.

Proposition 6 (existence of minimal typing). For each typable M , its minimal service typing Γ_0 exists. Further such Γ_0 is algorithmically calculable from M .

Theorem 7 (subject reduction). If $\Gamma \vdash N \triangleright \Delta$ and $N \rightarrow N'$ then $\Gamma \vdash N' \triangleright \Delta$.

Unlike the global calculus, the untyped end-point calculus can have communication error. Its absence is guaranteed by the type system. Let us say M has a *communication error* when:

$$M \equiv C[A[s \triangleright \Sigma_i \text{op}_i(x_i). P_i | R]_{\sigma'} | B[\overline{s} \triangleleft \text{op}(e). Q | S]_{\sigma'}]$$

where in both cases $\text{op} \notin \{\text{op}_i\}$ and $C[]$ is a reduction context (i.e. a context whose hole is not under a prefix). That is, M has a communication error when it contains an input and an output at a common channel which however do not match in operator names. A basic corollary of Theorem 7 follows.

Corollary 8 (lack of communication error). If $\Gamma \vdash N \triangleright \Delta$ and $N \rightarrow^* M$, then M never contains a communication error.

⁴ The direction of the subtyping is converse to (and consistent with) [13].

5 The End-Point Projection

This section establishes a formal link from the global calculus to the end-point calculus: a global description which conforms to the three properties, *connectedness*, *well-threadedness* and *coherency* can be mapped to the end-points preserving the three desirable properties, *type preservation*, and *soundness* and *completeness* of the operational semantics. Throughout we only consider well-typed terms for both the global and end-point calculi.

5.1 Connectedness

To define connectedness, we need to say which participant initiates an action in a given interaction I : this participant should be the place where the preceding event happens. First assume we annotate recursion variable with a participant name, e.g. $\mu X^A. I$ etc.

Definition 9 (initiating participants). Given a hiding-free interaction I , its initiating participants, denoted $\text{top}(I)$, is inductively given as follows:

$$\text{top}(I) \stackrel{\text{def}}{=} \begin{cases} \{A\} & \text{if } I \in Z \\ \emptyset & \text{if } I \equiv \mathbf{0} \\ \text{top}(I') & \text{if } I \equiv \mu X^A. I' \\ \text{top}(I_1) \cup \text{top}(I_2) & \text{if } I \equiv I_1 \mid I_2 \text{ or } I_1 + I_2 \end{cases}$$

where $Z = \{\text{if } e@A \text{ then } I_1 \text{ else } I_2, A \rightarrow B : \text{ch}(\nu \tilde{s}). I', A \rightarrow B : s(\text{op}, e, x). I, x@A := e. I', X^A\}$. If $A \in \text{top}(I)$, we say A is an initiating participant of I .

The map $\text{top}(I)$ generates a set of participants that initiates the first action of I . We can now present the definition of connectedness.

Definition 10 (connectedness). The collection of *connected interactions* Con is inductively generated as follows.

1. $\{\mathbf{0}, X^A\} \subseteq \text{Con}$.
2. $A \rightarrow B : \text{ch}(\nu s). I', A \rightarrow B : s(\text{op}, e, x). I', \mu X^B. I'$ and $x@B := e. I'$ are in Con if $I' \in \text{Con}$ and $\text{top}(I') = \{B\}$.
3. if $e@A$ then I_1 else I_2 , $I_1 + I_2$ and $I_1 \mid I_2$ are in Con if $I_1, I_2 \in \text{Con}$ and $\{A\} = \text{top}(I_1) = \text{top}(I_2)$.

Connectedness says that, in communication actions, only the message reception leads to activity (at the receiving participant), and that such activity should immediately follow the reception of messages. We note connectedness enjoys a subject reduction property, shared by well-threadedness and coherence.

5.2 Well-Threadedness

In order to formally introduce the notion of well-threadedness, we need to annotate a global interaction with threads.

Definition 11 (annotated interaction). *Annotated interactions*, denoted by $\mathcal{A}, \mathcal{A}', \dots$, are given by the following grammar.

$$\begin{aligned} \mathcal{A} ::= & A^{\tau_1} \rightarrow B^{\tau_2} : ch(\nu \bar{s}). \mathcal{A} \mid x@A^\tau := e. \mathcal{A} \mid \mathcal{A}_1 \mid^\tau \mathcal{A}_2 \\ & \mid A^{\tau_1} \rightarrow B^{\tau_2} : s\langle op, e, y \rangle. \mathcal{A} \mid \mu^\tau X^A. \mathcal{A} \mid \mathcal{A}_1 +^\tau \mathcal{A}_2 \\ & \mid \text{if } e@A^\tau \text{ then } \mathcal{A}_1 \text{ else } \mathcal{A}_2 \mid X_\tau^A \mid \mathbf{0} \end{aligned}$$

where $\tau_i \in \mathbb{N}$ (called *thread*) and $\tau_1 \neq \tau_2$ in the first two lines.

In the abstract syntax tree of the terms in the global calculus, each node is annotated with threads (given as natural numbers). The notions such as connectedness easily extend to annotated interactions. The following is an annotated interaction of a previous example:

$$\begin{aligned} & \text{Buyer}^1 \rightarrow \text{Seller}^2 : ch_1(\nu s). \text{Seller}^2 \rightarrow \text{Buyer}^3 : ch_2(\nu t). \\ & \text{Buyer}^3 \rightarrow \text{Seller}^2 : t\langle op_1, v_1, x \rangle. \\ & \text{Seller}^2 \rightarrow \text{Buyer}^1 : s\langle op_2, v_2, y \rangle \end{aligned}$$

Our task now is to find a notion of “consistent annotation” for annotated interactions, so that causality specified globally can be precisely realisable locally. For this purpose it is convenient to consider each \mathcal{A} as an inverted abstract syntax tree. Each node has a *constructor* which is annotated by either one thread or, if it is initiation or communication, an ordered pair of threads.

- Definition 12.**
1. If a node in \mathcal{A} is initialisation or communication from B to C and is annotated by (τ_1, τ_2) , then τ_1 (resp. τ_2) is the *active* (resp. *passive*) *thread* by B (resp. by C) of that node. If the node has other constructors, its annotating thread is both active and passive.
 2. If a node occurs (resp. directly) above some node, then the former is a (*resp. direct*) *predecessor* of the latter. Symmetrically we define (*direct*) *successor*.

Note if a node is a predecessor of another then the former execution should temporarily precedes that of the latter. We can now introduce the consistency condition for thread annotation. Below in (G2) we assume the bound name condition for session names.

Definition 13. An annotated connected interaction \mathcal{A} is *globally consistent* or simply *consistent* if the following conditions hold.

- (G1) **Freshness Condition:** For each node of \mathcal{A} , if it starts with initialisation, then its passive thread should be fresh w.r.t. all of its predecessors (if any).
- (G2) **Session Consistency:** If a node of \mathcal{A} starts with a communication between B and C via (say) s and another node of \mathcal{A} starts with a communication via s or an initialisation which opens s , then the thread by B (resp. by C) of the former node should coincide with the thread by B (resp. by C) of the latter node.
- (G3) **Causal Consistency:** If a node of \mathcal{A} is the direct successor of another node of \mathcal{A} (except when the latter node is a top-level parallel composition node), then the latter’s active thread should coincide with the former passive thread.

In (G3) we say a parallel composition is *top-level* if its predecessors (if any) only include parallel compositions and restrictions.

(G1) says a fresh thread starts when a service is invoked. (G2) says two distinct interactions in the same session (which are, by typing, always between the same pair of participants) should be given the same threads w.r.t. each participant. (G3) says if A has an input annotated as a (passive) thread then its immediately following output should be annotated by the same (but this time active) thread.

Below we say I has an annotation \mathcal{A} when removing all annotations from \mathcal{A} coincides with I .

Definition 14 (well-threaded interactions). I is *well-threaded* when it is connected and has a consistent annotation.

Note that well-threadedness implies connectedness (hence well-typedness). In Appendix C, we give a type discipline accepting all and only well-threaded interactions, from which we can derive a sound and complete inductive algorithm to check well-threadedness.

5.3 Coherence and End-Point Projection

We now define coherence and then end-point projection. First, we give the notion of mergeability of threads. In the rest of the paper, a *typed term* (in the end-point calculus) is a typed sequent $\Gamma \vdash_A P \triangleright \Delta$ or $\Gamma \vdash M \triangleright \Delta$. Moreover, a relation over typed processes or networks (in the end-point calculus) is *typed* if each related pair of typed terms have the same typing.

Definition 15 (mergeability). *Mergeability relation*, denoted \bowtie , is the smallest typed equivalence over terms up to \equiv closed under all typed contexts and the rule:

$$\frac{\forall i \in J \cap K. P_i \bowtie Q_i \quad \forall j \in J \setminus K, k \in K \setminus J. op_j \neq op_k}{s \triangleright \Sigma_{j \in J} op_j(x_j). P_j \bowtie s \triangleright \Sigma_{k \in K} op_k(x_k). Q_k}$$

When $P \bowtie Q$, we say P and Q are *mergeable*.

The relation \bowtie checks whether two given processes behave without contradicting when they come to the same course of interactions, i.e. when the same input branch is selected by the interacting party. Thus the rules above say that we can allow differences in input branches which do not overlap, but we do demand each pair of behaviours with the same operation to be identical.

Definition 16 (merge operator). \sqcup is a partial commutative binary operator on processes, such that (1) if P is a prefixed process with a service channel as its subject, then $P \sqcup \mathbf{0} = \mathbf{0} \sqcup P = P$; and (2) $s \triangleright \Sigma_{i \in J} op_i(y_i). P_i \sqcup s \triangleright \Sigma_{i \in K} op_i(y_i). Q_i$ is defined as:

$$\Sigma_{i \in J \cap K} op_i(y_i). (P_i \sqcup Q_i) + \Sigma_{i \in J \setminus K} op_i(y_i). P_i + \Sigma_{i \in K \setminus J} op_i(y_i). Q_i$$

where we assume that every time \sqcup is applied in the defining clause, say to P and Q , we have $P \bowtie Q$; and otherwise it is the identity. When these conditions are not satisfied, the operation is undefined.

Before introducing projection onto threads, we add a further annotation to each recursion and each recursion variable. Given $\mu^\tau X^A . \mathcal{A}$ in an annotated interaction, let $\{\tau_i\}$ be the set of threads occurring in, but *not* initiated in, \mathcal{A} . Then we further annotate this recursion as $\mu^{\tau:\{\tau_i\}} X$ and each free X_τ^A in \mathcal{A} as $X_{\tau:\{\tau_i\}}^A$. The added information is used for taking off unnecessary recursion from endpoint processes.⁵

Definition 17 (thread projection). Let \mathcal{A} be consistently annotated. Then the partial operation $\text{TP}(\mathcal{A}, \tau)$ is defined as follows:

- $\text{TP}(A^{\tau_1} \rightarrow B^{\tau_2} : b(\nu \tilde{s}), \mathcal{A}, \tau) \stackrel{\text{def}}{=} \begin{cases} \overline{b}(\nu \tilde{s}). \text{TP}(\mathcal{A}, \tau_1) & \text{if } \tau = \tau_1 \\ !b(\tilde{s}). \text{TP}(\mathcal{A}, \tau_2) & \text{if } \tau = \tau_2 \\ \text{TP}(\mathcal{A}, \tau) & \text{otherwise} \end{cases}$
- $\text{TP}(A^{\tau_1} \rightarrow B^{\tau_2} : s\langle \text{op}_i, e_i, x_i \rangle, \mathcal{A}, \tau) \stackrel{\text{def}}{=} \begin{cases} \overline{s} \triangleleft \text{op}(e), \text{TP}(\mathcal{A}, \tau) & \text{if } \tau = \tau_1 \\ s \triangleright \text{op}_i(x_i), \text{TP}(\mathcal{A}, \tau) & \text{if } \tau = \tau_2 \\ \text{TP}(\mathcal{A}, \tau) & \text{otherwise} \end{cases}$
- $\text{TP}(\text{if } e @ A^{\tau'} \text{ then } \mathcal{A}_1 \text{ else } \mathcal{A}_2, \tau) \stackrel{\text{def}}{=} \begin{cases} \text{if } e \text{ then } \text{TP}(\mathcal{A}_1, \tau') \text{ else } \text{TP}(\mathcal{A}_2, \tau') & \text{if } \tau = \tau' \\ \text{TP}(\mathcal{A}_1, \tau) \sqcup \text{TP}(\mathcal{A}_2, \tau) & \text{otherwise} \end{cases}$
- $\text{TP}(x @ A^{\tau'} := e, \mathcal{A}, \tau) \stackrel{\text{def}}{=} \begin{cases} x := e. \text{TP}(\mathcal{A}, \tau') & \text{if } \tau = \tau' \\ \text{TP}(\mathcal{A}, \tau) & \text{otherwise} \end{cases}$
- $\text{TP}(\mathcal{A}_1 \oplus^{\tau'} \mathcal{A}_2, \tau) \stackrel{\text{def}}{=} \begin{cases} \text{TP}(\mathcal{A}_1, \tau') \oplus \text{TP}(\mathcal{A}_2, \tau') & \text{if } \tau = \tau' \\ \text{TP}(\mathcal{A}_1, \tau) \sqcup \text{TP}(\mathcal{A}_2, \tau) & \text{otherwise} \end{cases}$
- $\text{TP}(\mathcal{A}_1 \upharpoonright^{\tau'} \mathcal{A}_2, \tau) \stackrel{\text{def}}{=} \text{TP}(\mathcal{A}_1, \tau') \upharpoonright \text{TP}(\mathcal{A}_2, \tau')$
- $\text{TP}(\mu^{\tau:\{\tilde{\tau}_i\}} X^A . \mathcal{A}, \tau) \stackrel{\text{def}}{=} \mu X. \text{TP}(\mathcal{A}, \tau)$ if $\tau \in \{\tilde{\tau}_i\}$, $\text{TP}(\mathcal{A}, \tau)$ if else.
- $\text{TP}(X_{\tau:\{\tilde{\tau}_i\}}^A, \tau) \stackrel{\text{def}}{=} X$ if $\tau \in \{\tilde{\tau}_i\}$, $\mathbf{0}$ if else.
- $\text{TP}(\mathbf{0}, \tau) \stackrel{\text{def}}{=} \mathbf{0}$

If $\text{TP}(\mathcal{A}, \tau)$ is undefined then we set $\text{TP}(\mathcal{A}, \tau) = \perp$.

Note the thread projection already uses the definedness of the \sqcup operator. The notion of coherence assumes this thread-level mergeability, extending it to inter-thread consistency. As noted in §2, the need for inter-thread consistency arises because the description of the behaviour of a service may as well be scattered over more than one places in a global description. Since each service channel ch uniquely defines a service, we can collect all threads contributing to its behaviour by taking the passive thread of each session initialisation at ch .

We now define coherence of well-threaded annotated interaction. Below, given an annotated interaction \mathcal{A} , we write $\tau_1 \equiv_{\mathcal{A}} \tau_2$ whenever τ_1 and τ_2 in \mathcal{A} belong to the same service channel.

⁵ The use of this added information does not affect behaviour, but is needed for type preservation. The added annotation is only used for thread projection.

Definition 18 (coherence). We say \mathcal{A} is *coherent* if it is consistently annotated (hence well-threaded) and satisfies:

1. For each thread τ in \mathcal{A} , $\text{TP}(\mathcal{A}, \tau)$ is well-defined.
2. For each pair of threads τ_1, τ_2 in \mathcal{A} with $\tau_1 \equiv_{\mathcal{A}} \tau_2$, we have $\text{TP}(\mathcal{A}, \tau_1) \bowtie \text{TP}(\mathcal{A}, \tau_2)$.

Proposition 19. *Given a well-typed I , it is decidable whether I is coherent (hence connected and well-threaded) or not.*

We can now define the endpoint projection. Below we call an interaction I *restriction-free* whenever it contains no terms of the form $(\nu s) I'$ as its subterm. Moreover, $[\tau]$ denotes the equivalence class ($\equiv_{\mathcal{A}}$) of τ and is formally defined in Appendix C.1

Definition 20 (end-point projection). Let I be a restriction-free and coherent interaction with free session names \bar{s} and let \mathcal{A} be one of its consistent annotations. Then the end point projection of $(\nu \bar{s}) \mathcal{A}$ under σ , denoted $\text{EPP}((\nu \bar{s}) \mathcal{A}, \sigma)$, is given as the following network.

$$(\nu \bar{s}) \prod_{A \in \text{part}(\mathcal{A})} A \left[\prod_{\tau \in [\tau]} \text{TP}(\mathcal{A}, \tau') \right]_{\sigma @ A}$$

where $\text{part}(\mathcal{A})$ denotes the set of participants mentioned in \mathcal{A} .

Remark. (Invariance of EPP w.r.t. Annotations) The result of the projection map defined above does *not* depend on a specific consistent annotation when I has no free session channels. See [11, Prop. 14, page 91] for details.

5.4 Pruning and Main Theorems

Suppose we have an interaction composed by two branches where the first two interactions are Buyer \rightarrow Seller : $ch(\nu s)$. Seller \rightarrow Buyer : $s\langle \text{ack} \rangle$ and then in one branch we have Buyer \rightarrow Seller : $s\langle \text{go} \rangle$ and in the other Buyer \rightarrow Seller : $s\langle \text{stop} \rangle$. We then get that Buyer and Seller are respectively projected as

$$\begin{aligned} & \overline{ch}(\nu s).s \triangleright \text{ack}().\bar{s} \triangleleft \text{go}() \oplus \overline{ch}(\nu s).s \triangleright \text{ack}().\bar{s} \triangleleft \text{stop}() \\ & !ch(s).\bar{s} \triangleleft \text{ack}().(s \triangleright \text{ok}() + s \triangleright \text{stop}()) \end{aligned}$$

By the dynamics of the choice operator, dropping one branch reduces to Seller \rightarrow Buyer : $s\langle \text{ack} \rangle$. Buyer \rightarrow Seller : $s\langle \text{go} \rangle$. Its end-point projection is the network:

$$\begin{array}{l} \text{Buyer} [\overline{ch}(\nu s).s \triangleright \text{ack}().\bar{s} \triangleleft \text{go}()]_{\sigma @ \text{Buyer}} \quad | \\ \text{Seller} [!ch(s).\bar{s} \triangleleft \text{ack}().s \triangleright \text{go}()]_{\sigma @ \text{Seller}} \end{array} \quad (2)$$

However the original end-point projection reduces as:

$$\begin{array}{l} \text{Buyer} [\overline{ch}(\nu s).s \triangleright \text{ack}().\bar{s} \triangleleft \text{go}()]_{\sigma @ \text{Buyer}} \quad | \\ \text{Seller} [!ch(s).\bar{s} \triangleleft \text{ack}().(s \triangleright \text{go}() \oplus s \triangleright \text{stop}())]_{\sigma @ \text{Seller}} \end{array} \quad (3)$$

There is discrepancy between (2) and (3): the former has *lost* one branch, while (3) keeps it. Notice this lost branch is inessential from the viewpoint of the internal dynamics of the configuration: “stop” is never used the global description obtained from reduction.

This example shows that a global interaction can lose information during reduction which is still kept in the corresponding reduction in its EPP, due to persistent behaviour at service channels. This motivates the introduction of the asymmetric relation of *pruning* that we shall use to state a property of the end-point projection. Below we write $!R$ when R is a n -fold composition of replications.

Definition 21 (pruning). Assume $\Gamma \vdash_A P \triangleright \Delta$, $\Gamma, \Gamma' \vdash_A Q \triangleright \Delta$ and, moreover, $\Gamma \vdash_A P \triangleright \Delta$ is a minimal typing. If further we have $Q \equiv Q_0 !R$ where $\Gamma \vdash Q_0 \triangleright \Delta$, $\Gamma' \vdash_A R$ and $P \bowtie Q_0$, then we write $\Gamma \vdash_A P < Q \triangleright \Delta$ or $P < Q$ for short; and say P *prunes* Q .

The pruning $P < Q$ indicates P is the result of cutting off “unnecessary branches” of Q , in the light of P ’s own typing. $<$ is in fact a typed strong bisimulation in the sense that $P < Q$ means they have precisely the same observable behaviours *except for the visible input actions at pruned inputs, either branches or replicated channels*. Thus in particular it satisfies the following condition.

Lemma 22 (pruning lemma). $<$ is a strong reduction bisimulation in the sense that it satisfies the following two clauses:

1. If $M < N$ and $M \rightarrow M'$ then $N \rightarrow N'$ such that $M' < N'$.
2. If $M < N$ and $N \rightarrow N'$ then $M \rightarrow M'$ such that $M' < N'$.

Further $<$ is transitive.

As noted, $<$ satisfies a stronger property of being a strong bisimulation w.r.t. typed transitions under the minimal typing of the l.h.s. processes. We have finally arrived at the main results of this paper. *Below and henceforth we use I, I', \dots to denote consistently annotated interactions.* \equiv_{rec} denotes the extension of \equiv with the folding/unfolding of recursion.

Theorem 23 (End-Point Projection). Assume

I is coherent. Assume further $\Gamma \vdash I \triangleright \Delta$ and $\Gamma \vdash \sigma$. Then we have:

1. (type preservation) If $\Gamma \vdash I \triangleright \Delta$ is the minimal typing of I , then $\Gamma \vdash \text{EPP}(I, \sigma) \triangleright \Delta'$ where Δ' is the result of replacing each occurrence of type assignment in Δ , say $\tilde{s}@A : \alpha$, with $\tilde{s} : \perp$. In particular, if $\Gamma \vdash I \triangleright \emptyset$ then $\Gamma \vdash \text{EPP}(I, \sigma) \triangleright \emptyset$.
2. (soundness) if $\text{EPP}(I, \sigma) \rightarrow N$ then there exists I' such that $(\sigma, I) \rightarrow (\sigma', I')$ and $\text{EPP}(I', \sigma') < \equiv_{rec} N$.
3. (completeness) If $(\sigma, I) \rightarrow (\sigma', I')$ then $\text{EPP}(I, \sigma) \rightarrow N$ such that $\text{EPP}(I', \sigma') < N$.

Proof Outline. For (1), type preservation, we first note the participants do not matter in the minimal typing for processes, i.e. we have $\Gamma \vdash \Pi_i A_i [P_i]_{\sigma_i} \triangleright \Delta$ iff $\Gamma \vdash \Pi_i P_i \triangleright \Delta$ as far as each σ_i is typable under Γ and assignment of channels to participants conform to Γ . Thus without loss of precision we only consider process typing from now on. We use the following auxiliary typing systems.

- The minimal typing system for annotated terms in the global calculus, with sequent $\Gamma \vdash_{\min} I \triangleright \Delta$.
- The per-thread minimal typing system for the same, with sequent $\Gamma \vdash_{\min}^{\tau} I \triangleright \Delta$, which assign the minimal typing to the specified thread τ of I .
- The minimal typing system for the endpoint calculus, with sequent $\Gamma \vdash_{\min} P \triangleright \Delta$.

These typing systems are given in [11, §10,12,16]. If a grouping of free session names in P is assumed, each of these systems determines the typing uniquely once a term (and a thread in the third case) is given. We therefore assume such a grouping (we can stipulate any appropriate one). We can then show the following.

- (a) $\Gamma \vdash_{\min} I \triangleright \Delta$ iff $\sqcup_i \Gamma_i \vdash_{\min} I \triangleright \sqcup_i \Delta_i$ where $\{\tau_i\}$ exhausts the threads in I and, for each τ_i , we have $\Gamma_i \vdash_{\min}^{\tau_i} I \triangleright \Delta_i$.
- (b) $\Gamma_i \vdash_{\min}^{\tau_i} I \triangleright \Delta_i$ iff $\Gamma_i \vdash_{\min} \text{TP}(I, \tau_i) \triangleright \Delta'_i$ where Δ'_i is the result of replacing each (e.g.) $\tilde{s}[A, B] : \alpha$ in Δ with $\tilde{s} : \perp$.
- (c) Given $\{P_i\}$, suppose $P_i \bowtie P_j$ for each $i, j \in I$. Then $\sqcup_i \Gamma_i \vdash_{\min} \sqcup_i P_i \triangleright \sqcup_i \Delta_i$.

where $\sqcup_i \Gamma_i$ and $\sqcup_i \Delta_i$ are the appropriate merge of typings, defined following the merge of processes. Now suppose $\Gamma \vdash_{\min} I \triangleright \Delta$ and $\{\tau_i\}$ are the threads in I . By (a) we have $\Gamma = \sqcup_i \Gamma_i$ and $\Delta = \sqcup_i \Delta_i$ such that $\Gamma_i \vdash_{\min}^{\tau_i} I \triangleright \Delta_i$. Hence by (b) we have $\Gamma_i \vdash_{\min} \text{TP}(I, \tau_i) \triangleright \Delta_i$ for each τ_i . Hence by (c) we have $\Gamma \vdash_{\min} \sqcup_i \text{TP}(I, \tau_i) \triangleright \Delta$, as required.

For (2) and (3), consider coherent $I = \Pi_i I_i$ and its projection:

$$M \stackrel{\text{def}}{=} A[\{P_l\}_{l \in L}]_{\sigma_A} \mid B[\{Q_m\}_{m \in M}]_{\sigma_B} \mid C[\{R_n\}_{n \in N}]_{\sigma_C}$$

Above for simplicity we consider only three participants, ignore hiding, and let each P_l etc. be a thread projection (the reasoning is similar in the general case).

For (2), soundness, assume $M \rightarrow M'$. By induction on the reduction rules, there is the corresponding redex in I . The rest is case analysis of the redex, taking the results of reducing I_i of I and either a thread (if it is not interaction) or a pair of threads (if it is). We then collect all threads again and compare the results. As a simple case, $P_l \stackrel{\text{def}}{=} x := e.P'_l$ results in P'_l with an altered state, while $I_i \stackrel{\text{def}}{=} x@A := e.I'_i$ results in I'_i with the same state. The projection of I'_i is the same as the projection of I_i except it loses $x := e$ from the corresponding thread, in this case P_l , that is we get P'_l . For all other cases it is possible the projection loses some branches or the whole replicated process, which we equate by $<$. Session initiation and recursion are the most interesting cases. If the redex is obtained by $P_l \stackrel{\text{def}}{=} !ch(\tilde{s}).P'_l$ and $Q_m \stackrel{\text{def}}{=} \overline{ch}(\nu\tilde{s}).Q'_m$ then we must have $I_i \stackrel{\text{def}}{=} A^1 \rightarrow B^2 : ch(\nu\tilde{s}).I'_i$ (with abuse of notation) which results in I'_i . The projection of the latter must have P'_l and Q'_m and may (or may not) have $!ch(\tilde{s}).P''_l$ (ch session initiation in other threads). The thesis for this case is then implied by the following two properties of pruning and merge operator:

- $P < P \sqcup Q$
- if $P \sqcup Q = !ch(\tilde{s}).R$ then $P = !ch(\tilde{s}).P'$ and $Q = !ch(\tilde{s}).Q'$ and $R = P' \sqcup Q'$.

In the recursion case, we have the redex $P_l \stackrel{\text{def}}{=} \mu X. P'_l$. It is important to observe that an unfolding for applying recursion in $EPP(I, \sigma)$ does not fully correspond to an unfolding in I . In fact an unfolding in the latter may imply many unfolding in $EPP(I, \sigma)$. Nevertheless, it is not difficult to prove that we can always unfold further in $EPP(I, \sigma)$ so to obtain the projection of unfolding in I . Then, we can prove this case by exploiting induction hypothesis.

(3), completeness, is by a similar reasoning. For details, see [11]. \square

By Corollary 8 and Lemma 22, Theorem 23 immediately implies:

Corollary 24. *In 2 below, we let $<_{rec} \stackrel{\text{def}}{=} (< \cup \equiv_{rec})^*$.*

1. (error freedom) *If $\Gamma \vdash I$ and $\Gamma \vdash \sigma$, then $EPP(I, \sigma)$ has no communication error.*
2. (soundness) *if $EPP(I, \sigma) \rightarrow^n N$ then there exists I' such that $(\sigma, I) \rightarrow^n (\sigma', I')$ and $EPP(I', \sigma') <_{rec} N$.*
3. (completeness) *If $(\sigma, I) \rightarrow^n (\sigma', I')$ then $EPP(I, \sigma) \rightarrow^n N$ such that $EPP(I', \sigma') < N$.*

Proof. (1) is immediate from Theorem 23 (1) and Corollary 8 (page 19). (2) and (3) are by Lemma 22 (1,2) and Theorem 23 (2, 3), combined with the standard tiling argument and induction on n . For example, for (2), the case when $n = 1$ is Theorem 23 (2). Suppose the statement holds up to n reductions and assume $EPP(\mathcal{A}, \sigma) \rightarrow^{n+1} N$. By definition this means $EPP(\mathcal{A}, \sigma) \rightarrow^n N_0 \rightarrow N$ for some N_0 . Hence by (IH) there exists \mathcal{A}'_0 such that $(\sigma, \mathcal{A}) \rightarrow^n (\sigma'_0, \mathcal{A}'_0)$ and $EPP(\mathcal{A}'_0, \sigma'_0) <_{rec} N_0$. By $N_0 \rightarrow N$ and since $<_{rec}$ immediately satisfies the same simulation property as $<$, $EPP(\mathcal{A}'_0, \sigma'_0) \rightarrow N''$ such that $N' <_{rec} N''$. By Theorem 23 (2) again we have $(\sigma'_0, \mathcal{A}'_0) \rightarrow (\sigma', \mathcal{A}')$ such that $EPP(\mathcal{A}', \sigma') < N''$. By the transitive of $<_{REC}$ we have $EPP(\mathcal{A}', \sigma') < N'$ as required. \square

6 Extensions and Applications of EPP Theory

6.1 Local variable declaration.

We consider extensions and applications of the theory of EPP. First, we augment the syntax of global/local calculi with one useful construct, *local variable declaration*:

$$\text{newvar } x@A := e \text{ in } I \quad \text{newvar } x := e \text{ in } P$$

This construct is indispensable especially for repeatedly invocable behaviours, i.e. those of services. Suppose a bookseller is invoked by two buyers simultaneously, each asking a quote for a different book. If these two threads share a variable, these two requests will get confused. The use of local variable declaration can avoid such confusion. The dynamics and typing of this construct are standard [26]. For endpoint projection, it is treated just as assignment.

6.2 Intra-Participant Interaction.

In §3.3, we demanded that, in the grammar of service typing, $A \neq B$ in $\delta[A, B]$. This means well-typed global terms never have an intra-participant interaction. This is a natural assumption in a business protocol which primarily specifies inter-organisational interactions: however it can be restrictive in other contexts. Under connectedness (whose definition does not change), we can easily adapt the EPP theory to the inclusion of intra-participant interactions. First, the typing rules in Table 2, page 13, takes off (G-TC I) and refines (G-TC II) so that the typing $\delta[A, B] : \alpha$ always reflects the direction of the interaction just inferred. This allows us to treat the case when A and B are equal. The key change is in well-threadedness. When $A = B$, the condition (G2) (session consistency) in Definition 13 is problematic since we do not know which of the two threads should be given to which participant. However stipulating the following condition solves this ambiguity:

Local Causal Consistency: If there is a downward sequence of actions which starts from an active thread τ and ends with an action in which τ occurs for the first time (i.e. τ occurs in no intermediate actions in the sequence), then the latter τ occurs passively.

We also note this condition is a *consequence* of (G1–3) in the theory without intra-participant interaction so that we are not adding any extra constraint to inter-participant interactions.

6.3 Name Passing.

An extension which is technically significant and practically useful is the introduction of *channel passing*. Channel passing is often essential in business protocols. As an example, consider the following refinement of Buyer-Seller Protocol.

Buyer wants to buy a hardware from Seller, but Buyer knows no Seller's address on the net, i.e. it does not know Seller's service channel. The only thing Buyer knows is a service channel hardware of a DirectoryService, which will send back the address of a Seller to Buyer which in turn interacts with that Seller through the obtained channel.

In such a situation, Buyer has no prior knowledge of not only the seller's channel but also the participant itself. In a global description including its typing, participant names play a basic role. Can we leave the name of a participant and its channels unknown and still have a consistent EPP theory? This has been an open problem left in WS-CDL's current specification (which allows channel passing only for a fixed participant). Below we restrict our attention to service channel passing, excluding session name passing (which poses an additional technical issue [15]).

First, at the level of the endpoint calculus, it suffices to use the channel passing in the standard π -calculus.

$$\overline{\text{DirectoryService}}(s).s(y).\bar{y}(t).P$$

which describes the initial behaviour of Buyer. Note y is an imperative variable, so that $y(t).P$ first *reads* the content of y then uses it for communication. The typing rules are extended accordingly.

In the global calculus, we introduce a syntactic variable Y , called a *participant placeholder*, for denoting anonymous participants. For example we can write:

$$A \rightarrow Y : x(\nu \tilde{s}). I \quad Y \rightarrow Y' : s\langle \text{op}, e, y \rangle. I$$

The newly added $A \rightarrow Y : x(\nu \tilde{s}). I$ intuitively says:

A starts a session with session names \tilde{s} on the service channel stored in x at the location A .

The participant at which the service is offered is left unknown by placing a placeholder Y . However this will be instantiated once the variable x at A is inspected. For example, if x is evaluated to $ch@B$ in the store, the interaction takes place as in $A \rightarrow B : ch(\nu \tilde{s}). I$.

As an example, we present the buyer-seller-directory scenario discussed above:

$$\begin{aligned} & \text{Buyer} \rightarrow \text{Directory} : \text{hardware}(\nu s). \\ & \text{Directory} \rightarrow \text{Buyer} : s\langle \text{sell}, \text{hware@amazon.co.uk}, x \rangle. \\ & (\text{Buyer} \rightarrow Y : x(\nu s'), Y \rightarrow \text{Buyer} : s'\langle \text{OK}, \text{data}, y \rangle \mid \\ & \text{Buyer} \rightarrow \text{Directory} : s\langle \text{more}, \text{""}, z \rangle. \\ & \text{Directory} \rightarrow \text{Buyer} : s\langle \text{sell}, \text{hardware@pcworld.co.uk}, x \rangle. \\ & \text{Buyer} \rightarrow Y' : x(\nu s''). Y' \rightarrow \text{Buyer} : s''\langle \text{OK}, \text{data}, y \rangle) \end{aligned}$$

Note that, depending on the channel sent from Directory, Y and Y' are assigned to different participants.

The dynamics of the global calculus adds the rule which infers:

$$(\sigma, A \rightarrow Y : x(\nu \tilde{s}). I) \rightarrow (\sigma, (\nu \tilde{s}) I[B/Y])$$

whenever we have $\sigma@A(x) = ch@B$.

For types, we first extend the basic types θ with $(\tilde{s})\alpha$. We then add, with the obvious extension to the syntax of types:

$$\frac{\Gamma \vdash x@W_1 : (\tilde{s})\alpha \quad \Gamma \vdash I \triangleright \Delta \cdot \tilde{s}[W_2, W_1] : \alpha}{\Gamma \vdash W_1 \rightarrow W_2 : x(\nu \tilde{s}). I \triangleright \Delta}$$

Other typing rules can be extended to deal with terms containing the participant variable Y in the same manner.

Finally, for the EPP theory, we need no change in the notion of connectedness. For well-threadedness, we first annotate placeholders regarding, e.g. $A \rightarrow Y : x(\nu \tilde{s}). I$ as the start of a new thread for Y , so we annotate it as $A^{\tau_1} \rightarrow Y^{\tau_2} : x(\nu \tilde{s}). I$ with τ_2 fresh. The definition of well-threadedness remains the same. Coherence however needs additional consideration. The variable $x@A$ can store different channels from different participants. For this purpose we use a typing system which records a possible set of assignment, in the shape $x@W_1 : C$ where C is a set of channels which may be instantiated into C . If some concrete channel is in C , the behaviour of that channel becomes

constrained by coherence. This set C is inferred, starting from some fixed set, by adding ch (as in $x@W_1 : C \cup \{ch@B\}$) when we infer, e.g. $W_1 \rightarrow W_2 : s\langle op_j, ch@B, x \rangle$. I , where W_i can be either of participants or placeholders.

Leaving the technical details to [11], we give a flavour of how this extension works by the end-point projection of the example above. We first consider the annotated interaction for placeholders.

$$\text{Buyer}^1 \rightarrow Y^3 : x(\nu s'). Y^3 \rightarrow \text{Buyer}^1 : s'\langle \text{OK}, \text{data}, y \rangle$$

In the projection of this thread, we have placed a hole – which should be substituted with the appropriate service channels.

$$\text{TP}(\mathcal{A}, 3) = !_-(s'). \overline{s'} \triangleleft \text{OK}\langle \text{data} \rangle$$

Thus, checking coherence consists in updating the definition of the function threads which induces the thread equivalence classes. But what equivalence classes should threads 3 and 4 belong to? We can use the prediction of all the possible values x can assume at runtime, i.e. `hware@amazon.co.uk` and `hardware@pcworld.co.uk`. We have to make sure that thread 3 belongs to both $\text{threads}(\mathcal{A}, \text{hware})$ and $\text{threads}(\mathcal{A}, \text{hardware})$. Then, if we are end-point projecting in `amazon.co.uk` we will substitute `hware` to `_` in both thread projections, and if we are end-point projecting `pcworld.co.uk` we will substitute `hardware` instead.

6.4 Conformance.

By relating global descriptions to their local counterpart, the presented theory allows us to make the best of the rich results from the study of process calculi. One such application is *conformance checking* (and its dynamic variant, runtime monitoring), discussed in Section 1. Our purpose is to have a formal criteria to say the communication behaviour of a program P conforms to a global specification I .

In process algebras, conformance checking verifies whether the behaviour of a process follows that of a specification, the latter also given as (or representable as) a process. As a theoretical basis of the notion of conformance, we can use behavioural theories such as (inverse of) simulation or bisimulation. In the present context, our purpose is to verify whether an implemented system, say P , conforms to a well-typed description in the global calculus. Through the use of the end-point projection, we can reduce this problem to the conformance between endpoint processes, allowing us to use the standard theory. We illustrate one basic instance in the following.

The conformance checking we consider starts from the use of type information. Let I be a global description and A be a participant whose behaviour we are interested in. Let S be an EPP of I onto A . Let P be a program which is supposedly implements the specification I at the participant A . Since the end-point projection generates a process whose typing coincides with that of the original global description, we can first check whether P can be typed under the minimal typing associated with S . If we cannot, then P does not conform to I .

Once we know the typing of P conforms to that of S , we can proceed to validation of behavioural conformance. As an example, let P be given by:

$$\overline{\text{QuoteCh}(v.s)}. s \triangleright \text{Quote}(x). \\ \text{if } (x \leq 100) \text{ then } \bar{s} \triangleleft \text{Accept}(\langle \rangle) \text{ else } \bar{s} \triangleleft \text{Reject}(\langle \rangle)$$

which may be used in the following configuration:

$$\text{Buyer}[P] \mid \text{Seller}[\! \text{QuoteCh}(s). \bar{s} \triangleleft \text{Quote}(300). \\ s \triangleright (\text{Accept}() + \text{Reject}() + \text{Restart}())]$$

Suppose we wish to check whether this P , as the behaviour of Buyer, conforms to a global specification given as follows.

$$\text{Buyer} \rightarrow \text{Seller} : \text{QuoteCh}(v.s). \\ \text{Seller} \rightarrow \text{Buyer} : s\langle \text{Quote}, 300, x \rangle. \\ \text{Buyer} \rightarrow \text{Seller} : s\langle \text{Accept} \rangle + \text{Buyer} \rightarrow \text{Seller} : s\langle \text{Reject} \rangle$$

We have already seen the end point projection of the specification above is given as follows.

$$\text{Buyer}[\overline{\text{QuoteCh}(v.s)}. s \triangleright \text{Quote}(x). \\ (\bar{s} \triangleleft \text{Accept}(\langle \rangle) \oplus \bar{s} \triangleleft \text{Reject}(\langle \rangle))] \mid \\ \text{Seller}[\! \text{QuoteCh}(s). \bar{s} \triangleleft \text{Quote}(300). \\ s \triangleright (\text{Accept}() + \text{Reject}())]$$

Let the endpoint process inhabiting Buyer above to be S . First, we can check the typability quite easily, in the sense that P is typable under the minimal typing of S . For example, the minimal type for the channel QuoteCh in S is given as

$$s \triangleleft \text{Quote}(\text{int}). s \triangleright (\text{Accept}(\text{null}) + \text{Reject}(\text{null}))$$

Call this type α . Then α is *not* the minimal type of P at the same channel, but it is indeed a type assignable to ch .

Second, on the basis of well-typedness, we check the conformance of behaviour using a typed transition relation. The transition we consider has the sequent of the form:

$$(\Gamma \vdash (Q, \sigma) \triangleright \Delta) \xrightarrow{l} (\Gamma' \vdash (Q', \sigma') \triangleright \Delta')$$

Following the framework in [5, 37], \xrightarrow{l} is *typed* in the sense that we only consider those actions deemed possible by Γ and Δ . In particular, if Γ or Δ specifies input branches which are less than Q , then inputs at the extra branches do not take place. We write $\xrightarrow{\hat{l}}$ for the standard weak transition abstracting the silent transition. We can then define the following conformance relation (we include typability: as always we fix an appropriate grouping of free session channels if any).

We say a type relation \mathcal{R} over typed terms in the endpoint calculus is a *weak conformance* iff, whenever $\Gamma \vdash P_0 \mathcal{R} S_0 \triangleright \Delta$, we have: (1) Γ and Δ are the minimal typings of S ; and (2) $(\Gamma \vdash P_0 \triangleright \Delta) \xrightarrow{l} (\Gamma' \vdash P'_0 \triangleright \Delta')$ implies $(\Gamma \vdash S_0 \triangleright \Delta) \xrightarrow{\hat{l}} (\Gamma' \vdash S'_0 \triangleright \Delta')$ such that $\Gamma \vdash P'_0 \mathcal{R} S'_0 \triangleright \Delta$. If $\Gamma \vdash P_0 \mathcal{R} S_0$ for some weak conformance \mathcal{R} , we say P_0 *conforms to* S_0 .

Thus (apart from typability) the conformance of P_0 to S_0 says that all visible behaviours of P_0 are within what is specified in S_0 . We can indeed check that, for P and S above, P conforms to S in this sense. Thus this conformance is about safety: depending on the application needs, we may as well use a more stringent notion of conformance.

In summary, let I be a global description consisting of the participant A as well as other participants. Suppose P is a program which implements A 's behaviour. Then we can check the conformance of P against the specification I by projecting I to A , which we call S , and check P conforms to S , using the conformance relation noted above. The conformance of P to S may be checked through either hand-calculation (coinduction), model checking, mechanical syntactic approximation, or runtime monitoring.

7 Conclusions

This paper introduced a new formalism based on global description of communication behaviour, and the corresponding applied π -calculus. Both calculi are based on a new extension of session types, which can handle parallel interaction in one session. A theory of endpoint projection is developed, giving the three well-structuredness conditions on global descriptions. The sound and complete mapping from them to the corresponding endpoint processes is established.

Global descriptions have been practiced in various engineering contexts for a long time: the present work is a trial to realise its potential as a general programming method, centring on type structures for communication and the end-point projection. The EPP theory needs be further explored for all basic concurrent programming primitives, including general sequencing, various mutual exclusion operations, exceptions, timeout and other useful primitives. While channel passing in our language can encode a synchronisation mechanism, a valuable future topic is its interaction with primitives for locking primitives and software transaction memory, since the notion of atomicity undergoes a fundamental change when we move to communication-centered programming.

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A An Example of Endpoint Projection

In the following we illustrate the formal notion of endpoint projection we have developed in the paper using a fairly large toy example involving five participants. First, we explain the example in English; then we introduce the description in the global calculus; finally we project the description to endpoint processes.

A.1 Global Description in English

The example is an extension of the buyer-seller example introduced in section 2. The participants involved in this protocol are

1. Buyer (B)
2. Seller (S)
3. Vendor (V)
4. CreditChecker (CC)
5. RoyalMail (RM)

The protocol proceeds as follows:

1. Buyer requests a service ch_{CC} for company check to the credit checker CreditChecker by sending its name.
2. At this point CreditChecker can either give a positive or negative answer.
3. If the answer is positive:
 - (a) Buyer asks Seller for a quote about product $prod$;
 - (b) Seller then asks Vendor for service ch_V
 - (c) Seller starts recursion and asks Vendor for a quote about product $prod$;
 - (d) Vendor replies with a quote $quote$;
 - (e) Seller forwards $quote$ to Buyer increasing it by 10 units ($quote+10$);
 - (f) if the quote is reasonable ($reasonable(quote + 10)$) then:
 - i. Buyer sends Seller a confirmation ($quoteOK$) together with the credit ($cred$);
 - ii. Seller then contacts CreditChecker for checking the credit;
 - iii. If the credit is good then:
 - A. Seller contacts Shipper (service ch_{Sh});
 - B. Seller sends the delivery address;
 - C. Shipper sends a confirmation;
 - D. Seller forwards confirmation to Buyer;
 - iv. If the credit is bad:
 - A. CreditChecker tells Buyer;
 - B. Buyer tells Seller terminating the protocol;
 - (g) if the quote is not reasonable the protocol goes back to point 3c;
4. If the answer is negative then the protocol terminates.

A.2 Global Description in the Calculus

The global description consists of several components for readability. We directly give annotated interaction. The main description is:

1. $B^1 \rightarrow CC^2 : ch_{CC}(vs). CC^2 \rightarrow B^1 : s\langle ack \rangle.$
2. $B^1 \rightarrow CC^2 : s\langle companyCheck, sellerName, compName \rangle.$
3. {
4. $CC^2 \rightarrow B^1 : s\langle good \rangle. I_{good}$
5. +
6. $CC^2 \rightarrow B^1 : s\langle bad \rangle. \mathbf{0}$
7. }

where I_{good} in Line 4 is:

1. $B^1 \rightarrow S^3 : ch_S(vt). S^3 \rightarrow B^1 : r\langle ack \rangle.$
2. $B^1 \rightarrow S^3 : t\langle quoteReq, prod, prod \rangle.$
3. $S^3 \rightarrow V^4 : ch_V(vr).$
4. $V^4 \rightarrow S^3 : r\langle ack \rangle.$
5. $\mu X^3. \{$
6. $S^3 \rightarrow V^4 : r\langle quoteReq, prod, prod \rangle.$
7. $V^4 \rightarrow S^3 : r\langle quoteRes, quote, quote \rangle.$
8. $S^3 \rightarrow B^1 : t\langle quoteRes, quote + 10, quote \rangle.$
9. if $reasonable(quote)@B^1$ then
10. $B^1 \rightarrow S^3 : t\langle quoteOK, cred, cred \rangle.$
11. $S^3 \rightarrow CC^5 : ch_{CC}(vu).$
12. $CC^5 \rightarrow S^3 : u\langle ack \rangle.$
13. $S^3 \rightarrow CC^5 : u\langle personalCreditCheck, cred:adr, cred:adr \rangle.$
14. {
15. $CC^5 \rightarrow S^3 : u\langle good \rangle. I'_{good}$
16. +
17. $CC^5 \rightarrow S^3 : u\langle bad \rangle.$
18. $S^3 \rightarrow B^1 : t\langle yourCreditsBad \rangle. \mathbf{0}$
19. }
20. else $B^1 \rightarrow S^3 : t\langle quoteNotOK \rangle. X^3$
21. }

where I'_{good} in Line 15 is:

1. $S^3 \rightarrow R^6 : ch_R(vp).$
2. $R^6 \rightarrow S^3 : p\langle ack \rangle.$
3. $S^3 \rightarrow R^6 : p\langle deliv, adr, adr \rangle.$
4. $R^6 \rightarrow S^3 : p\langle conf \rangle.$
5. $S^3 \rightarrow B^1 : t\langle conf \rangle. \mathbf{0}$

We can check these descriptions are typable, strongly connected, well-threaded and coherent. For connectedness, the description given above uses a lot of acks. As we discussed in the long version, many of these acks are in fact unnecessary by using a relaxed notion of connectedness.

A.3 End-Point Projection of the Global Interaction

Following the definition of EPP in the paper, we first project the global description onto each thread. The first one is Buyer's only thread.

$$\begin{aligned}
 TP(I, 1) = & \overline{ch}_{CC}(vs). s \triangleright \text{ack}(). \bar{s} \triangleleft \text{companyCheck}\langle \text{sellerName} \rangle. \\
 & \{ s \triangleright \text{good}(). \overline{ch}_S(vt). t \triangleright \text{ack}(). \bar{t} \triangleleft \text{quoteReq}\langle \text{prod} \rangle. \\
 & \mu X. t \triangleright \text{quoteRes}\langle \text{quote} \rangle. \\
 & \quad \text{if } \text{reasonable}\langle \text{quote} \rangle \text{ then } \bar{t} \triangleleft \text{quoteOK}\langle \text{cred} \rangle. \\
 & \quad \quad \quad \{ t \triangleright \text{yourCreditsBad}() \\
 & \quad \quad \quad + \\
 & \quad \quad \quad t \triangleright \text{conf}() \} \\
 & \quad \text{else } \bar{t} \triangleleft \text{quoteNoteOK}\langle \rangle. X \\
 & + \\
 & s \triangleright \text{bad}(). \mathbf{0} \}
 \end{aligned}$$

Note this thread starts before the recursion and go through inside the (global) recursion. Thus the projected endpoint behaviour also contains recursion.

The next projection is onto the first thread of CreditChecker (note this participant has two threads, 2 and 5).

$$\begin{aligned}
 TP(I, 2) = & !ch_{CC}(s). \bar{s} \triangleleft \text{ack}\langle \rangle. s \triangleright \text{companyCheck}\langle \text{compName} \rangle. \\
 & \{ \bar{s} \triangleleft \text{good}\langle \rangle. \\
 & \oplus \\
 & \bar{s} \triangleleft \text{bad}\langle \rangle. \mathbf{0} \}
 \end{aligned}$$

Note no recursion is involved in this thread projection, simply because the thread 2 does not occur inside the recursion.

Next we jump to Thread 5, which is another component of CreditChecker.

$$\begin{aligned}
 TP(I, 5) = & !ch_{CC}(u). \bar{u} \triangleleft \text{ack}\langle \rangle. u \triangleright \text{personalCreditCheck}\langle \text{cred}:\text{adr} \rangle \\
 & (\bar{u} \triangleleft \text{good}\langle \rangle) \\
 & + \\
 & \bar{u} \triangleleft \text{bad}\langle \rangle
 \end{aligned}$$

Note the process does not include the recursion either. This is because it is inside a recursion and it initiates a new thread there. As a result the code is identical with the projection onto Thread 2.

We now move to the projection onto the unique thread of Seller, which is Thread 3.

$$\begin{aligned}
TP(I, 3) = & !ch_S(t). \bar{t} \triangleleft \text{ack}\langle \rangle. t \triangleright \text{quoteReq}(\text{prod}). \overline{ch_V}(vr). t \triangleright \text{ack}\langle \rangle. \\
& \mu X. \bar{r} \triangleleft \text{quoteReq}(\text{prod}). r \triangleright \text{quoteRes}(\text{quote}). \\
& \bar{t} \triangleleft \text{quoteRes}(\text{quote} + 10). \\
& \{ t \triangleright \text{quoteOK}(\text{cred}). \overline{ch_C}(vu). u \triangleright \text{ack}\langle \rangle. \\
& \bar{u} \triangleleft \text{personalCreditCheck}(\text{cred}:\text{adr}). \\
& \quad \{ u \triangleright \text{good}\langle \rangle. \overline{ch_R}(vp). p \triangleright \text{ack}\langle \rangle \\
& \quad \bar{p} \triangleleft \text{deliv}(\text{adr}). p \triangleright \text{conf}\langle \bar{t} \triangleleft \text{conf}\langle \rangle \\
& \quad + \\
& \quad u \triangleright \text{bad}\langle \rangle. \bar{t} \triangleleft \text{CreditsBad}\langle \rangle \} \\
& + \\
& t \triangleright \text{quoteNoteOK}\langle \rangle. X
\end{aligned}$$

As before, this thread starts outside of the recursion in the global description and is also used inside, so that both the recursion and the recursion variable are used as they are, leading to the recursive behaviour of the process. Note how the use of session functions as a way to handle recursion appropriately in EPP.

The projection onto the unique thread of Vendor follows.

$$\begin{aligned}
TP(I, 4) = & !ch_V(r). \bar{t} \triangleleft \text{ack}\langle \rangle. \\
& \mu X. r \triangleright \text{quoteReq}(\text{prod}). \bar{r} \triangleleft \text{QuoteRes}(\text{quote}). X
\end{aligned}$$

Finally we end with the projection onto Thread 6, giving the simple behaviour of RoyalMail.

$$TP(I, 6) = !ch_R(p). \bar{p} \triangleleft \text{ack}\langle \rangle. p \triangleright \text{deliv}(\text{adr}). \bar{p} \triangleleft \text{conf}\langle \rangle$$

As before, Thread 6 does not contain recursion since it is fully inside the (global) recursion, initiating a thread there.

As noted, there are two threads (2 and 5) that belong to the same class of equivalence i.e. they are part of the same service channel ch_{CC} . This means that we must merge the two threads in the final EPP. By applying the merge operator, and noting they are evidently mergeable, we get the following process:

$$\begin{aligned}
& !ch_{CC}(u). \bar{u} \triangleleft \text{ack}\langle \rangle. \\
& u \triangleright \left(\begin{array}{l} \text{personalCreditCheck}(\text{cred}:\text{adr}). (\bar{u} \triangleleft \text{good}\langle \rangle \oplus \text{bad}\langle \rangle) \\ + \\ \text{companyCheck}(\text{compName}). (\bar{u} \triangleleft \text{good}\langle \rangle \oplus \text{bad}\langle \rangle) \end{array} \right)
\end{aligned}$$

By which we have arrived at the endpoint behaviours of all participants realising the original global description.

The projection works because of the linear usage of channels inside each session and service channel principle, as well as the three well-structuredness conditions. We

Table 5 Semantics of the End-Point Calculus

$$\begin{array}{l}
 \text{(EP-I) } \frac{}{A[!ch(\bar{s}). P | P']_{\sigma} | B[\overline{ch}(v\bar{s}). Q | Q']_{\sigma'} \rightarrow (v\bar{s}) (A[!ch(\bar{s}). P | P']_{\sigma} | B[Q | Q']_{\sigma'})} \\
 \text{(EP-C) } \frac{\sigma \vdash e \Downarrow v}{A[s \triangleright \Sigma_i \text{op}_i(x_i). P_i | P']_{\sigma} | B[\bar{s} \triangleleft \text{op}_i \langle e \rangle. Q | Q']_{\sigma'} \rightarrow A[P_j | P']_{\sigma[x \mapsto v_j]} | B[Q | Q']_{\sigma'}} \\
 \text{(EP-I T) } \frac{\sigma \vdash e \Downarrow \text{tt}}{A[\text{if } e \text{ then } P_1 \text{ else } P_2 | P']_{\sigma} \rightarrow A[P_1 | P']_{\sigma}} \quad \text{(EP-P 1) } \frac{M \rightarrow M'}{M|N \rightarrow M'|N} \\
 \text{(EP-I F) } \frac{\sigma \vdash e \Downarrow \text{ff}}{A[\text{if } e \text{ then } P_1 \text{ else } P_2 | P']_{\sigma} \rightarrow A[P_2 | P']_{\sigma}} \quad \text{(EP-S) } \frac{i \in \{1, 2\}}{A[P_1 \oplus P_2 | R]_{\sigma} \rightarrow A[P_i | R]_{\sigma}} \\
 \text{(EP-R 1) } \frac{A[P]_{\sigma} \rightarrow A[P']_{\sigma'}}{A[(v\bar{s}) P]_{\sigma} \rightarrow A[(v\bar{s}) P']_{\sigma'}} \quad \text{(EP-A) } \frac{\sigma \vdash e \Downarrow v}{A[x := e. P | P']_{\sigma} \rightarrow A[P | P']_{\sigma[x \mapsto v]}} \\
 \text{(EP-R 2) } \frac{M \rightarrow M'}{(v\bar{s}) M \rightarrow (v\bar{s}) M'} \quad \text{(EP-S) } \frac{M \equiv M' \quad M' \rightarrow N' \quad N' \equiv N}{M \rightarrow N} \\
 \text{(EP-R) } \frac{A[P[\mu X.P/X] | Q]_{\sigma} | N \rightarrow N'}{A[\mu X.P | Q]_{\sigma} | N \rightarrow N'} \quad \text{(EP-P 2) } \frac{A[P_1 | R]_{\sigma} \rightarrow A[P'_1 | R]_{\sigma'}}{A[P_1 | P_2 | R]_{\sigma} \rightarrow A[P'_1 | P_2 | R]_{\sigma'}}
 \end{array}$$

believe many business protocols conform to these conditions (modulo relaxation of connectedness we discussed in the long version). How these conditions can be extended in disciplined ways to allow more “untamed” protocols (such as those involving exceptions) to be treated in the theory, is an interesting subject of further studies.

B Appendix: the End-Point Calculus

This appendix lists the full reduction rules and typing system of the end-point calculus.

The full typing rules are listed in Tables 7 and 6. The rule (EP-TB) for input in-session communication involves branching with distinct operators: the typing can have less branches than the real process, so that the process is prepared to receive any operator specified in the type. Rule (EP-TS) is its dual: the typing can have more branches than the real process, so that the process invokes with operators at most those specified in types. Combining (EP-TB) and (EP-TS), an output never tries to invoke a non-existent option in its matching input.

Rule (EP-TS) is for typing the inputting side of initialisation. Note we do not allow those session channels other than the target of initialisation to be present as the session typing in the premise: this prevents *free* session channels to be under the repli-

Table 6 Session Types for Processes in the End-Point Calculus

$$\begin{array}{c}
 \text{(EP-TB)} \quad \frac{K \subseteq J \quad s \in \tilde{s} \quad \Gamma \vdash x_j : \theta_j \quad \Gamma \vdash_A P_j \triangleright \Delta \cdot \tilde{s}@A : \alpha_j}{\Gamma \vdash s \triangleright \sum_{i \in J} \text{op}_i(x_i). P_i \triangleright \Delta \cdot \tilde{s}@A : s \blacktriangleright \sum_{i \in K} \text{op}_i(\theta_i). \alpha_i} \\
 \\
 \text{(EP-TS)} \quad \frac{j \in J \subseteq K \quad \Gamma \vdash e : \theta_j \quad \Gamma \vdash_A P \triangleright \Delta \cdot \tilde{s}@A : \alpha_j}{\Gamma \vdash_A \bar{s} \triangleleft \text{op}_j(e). P \triangleright \Delta \cdot \tilde{s}@A : s \blacktriangleleft \sum_{i \in K} \text{op}_i(\theta_i). \alpha_i} \\
 \\
 \text{(EP-TR 1)} \quad \frac{\Gamma \vdash_A P \triangleright \Delta, \tilde{s}_1 s \tilde{s}_2 : \perp}{\Gamma \vdash_A (\nu s) P \triangleright \Delta, \tilde{s}_1 \tilde{s}_2 : \perp} \qquad \text{(EP-TR 2)} \quad \frac{\Gamma \vdash_A P \triangleright \Delta, \varepsilon : \perp}{\Gamma \vdash_A (\nu s) P \triangleright \Delta} \\
 \\
 \text{(EP-TS)} \quad \frac{\Gamma \vdash_A P \triangleright \tilde{s}@A : \alpha}{\Gamma, ch@A : (\tilde{s})\alpha \vdash_A !ch(\tilde{s}). P \triangleright \emptyset} \qquad \text{(EP-TR)} \quad \frac{\Gamma, \overline{ch}@B : (\tilde{s})\alpha \vdash_A P \triangleright \Delta \cdot \tilde{s}@A : \alpha}{\Gamma, \overline{ch}@B : (\tilde{s})\alpha \vdash_A \overline{ch}(\nu \tilde{s}). P \triangleright \Delta} \\
 \\
 \text{(EP-TP)} \quad \frac{\Gamma \vdash_A P_i \triangleright \Delta_i \quad \Delta_1 \asymp \Delta_2}{\Gamma \vdash_A P_1 \mid Q_2 \triangleright \Delta_1 \odot \Delta_2} \qquad \text{(EP-TI)} \quad \frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash_A P_i \triangleright \Delta}{\Gamma \vdash_A \text{if } e \text{ then } P_1 \text{ else } P_2 \triangleright \Delta} \\
 \\
 \text{(EP-TA)} \quad \frac{\Gamma \vdash_A x : \theta \quad \Gamma \vdash e : \theta \quad \Gamma \vdash_A P \triangleright \Delta}{\Gamma \vdash_A x := e. P \triangleright \Delta} \qquad \text{(EP-TR)} \quad \frac{\Gamma, X : \Delta \vdash_A P \triangleright \Delta}{\Gamma \vdash_A \mu X. P \triangleright \Delta} \\
 \\
 \text{(EP-TS)} \quad \frac{\Gamma \vdash_A P \triangleright \Delta \quad \Gamma \vdash_A Q \triangleright \Delta}{\Gamma \vdash_A P \oplus Q \triangleright \Delta} \qquad \text{(EP-TV)} \quad \frac{-}{\Gamma, X : \Delta \vdash_A X \triangleright \Delta} \\
 \\
 \text{(EP-TB)} \quad \frac{\Gamma \vdash_A P \triangleright \Delta \quad \{\tilde{s}\} \cap \text{fsc}(\Delta) = \emptyset}{\Gamma \vdash_A P \triangleright \Delta \cdot \tilde{s} : \perp} \qquad \text{(EP-TI)} \quad \frac{\Gamma \text{ well-formed.}}{\Gamma \vdash_A \mathbf{0} \triangleright \emptyset} \\
 \\
 \text{(EP-TE)} \quad \frac{\Gamma \vdash_A P \triangleright \Delta \quad \{\tilde{s}\} \cap \text{fsc}(\Delta) = \emptyset}{\Gamma \vdash_A P \triangleright \Delta \cdot \tilde{s}@A : \text{end}}
 \end{array}$$

cated input, guaranteeing their linear usage. The typing in the conclusion means (by our convention) that ch or \overline{ch} does not occur in Γ . The outputting side of initialisation (rule (EP-TR)) is analogous, except that the linearity constraint needs not be specified. We assume that A and B are not identical. The fact we allow $\overline{ch}@B : (\tilde{s})\alpha$ to occur in the premise means an invocation to a service can be done as many times as needed (as far as it is type correct).

As for rule (EP-TP) and (EP-TP N) we need to define the operators \asymp and \odot .

1. (compatibility, 1) We write $\Gamma_1 \asymp \Gamma_2$ iff whenever ch occurs in both Γ_1 and Γ_2 , we have: (1) $ch@A : (\tilde{s})\alpha \in \Gamma_1$ and $ch@A : (\tilde{s})\bar{\alpha} \in \Gamma_2$ or its symmetric case; or (2) $ch@A : (\tilde{s})\alpha \in \Gamma_{1,2}$; similarly, for each X and x , we demand they have precisely the same assignments when occurring in both.

Table 7 Session Types for Networks in the End-Point Calculus

$$\begin{array}{c}
 \text{(EP-TP)} \quad \frac{\Gamma \vdash_A P \triangleright \Delta \quad \Gamma \vdash \sigma @ A}{\Gamma \vdash A[P]_{\sigma} \triangleright \Delta} \qquad \text{(EP-TP N)} \quad \frac{\Gamma \vdash N_1 \triangleright \Delta_1 \quad \Gamma \vdash N_2 \triangleright \Delta_2 \quad \Delta_1 \asymp \Delta_2}{\Gamma \vdash N_1 \mid N_2 \triangleright \Delta_1 \odot \Delta_2} \\
 \\
 \text{(EP-TE N)} \quad \frac{\Gamma \vdash M \triangleright \Delta \quad \{\tilde{s}\} \cap \text{fsc}(\Delta) = \emptyset}{\Gamma \vdash M \triangleright \Delta \cdot \tilde{s} @ A : \text{end}} \qquad \text{(EP-TR N1)} \quad \frac{\Gamma \vdash M \triangleright \Delta, \tilde{s}_1 \tilde{s}_2 : \perp}{\Gamma \vdash (\nu s) M \triangleright \Delta, \tilde{s}_1 \tilde{s}_2 : \perp} \\
 \\
 \text{(EP-TB N)} \quad \frac{\Gamma \vdash M \triangleright \Delta \quad \{\tilde{s}\} \cap \text{fsc}(\Delta) = \emptyset}{\Gamma \vdash M \triangleright \Delta \cdot \tilde{s} : \perp} \qquad \text{(EP-TR N2)} \quad \frac{\Gamma \vdash M \triangleright \Delta, \varepsilon : \perp}{\Gamma \vdash M \triangleright \Delta} \\
 \\
 \text{(EP-TI N)} \quad \frac{\{\tilde{s}_i\} \cap \{\tilde{s}_j\} = \emptyset \ (i \neq j)}{\Gamma \vdash \epsilon \triangleright \emptyset}
 \end{array}$$

2. (composition, 1) Suppose $\Gamma_1 \asymp \Gamma_2$. Then $\Gamma_1 \odot \Gamma_2$ is given as follows: (1) $ch @ A : (\tilde{s})\alpha \in \Gamma_1 \odot \Gamma_2$ iff $ch @ A : (\tilde{s})\alpha \in \Gamma_i$. (2) $\overline{ch} @ A : (\tilde{s})\alpha \in \Gamma_1 \odot \Gamma_2$ iff $\overline{ch} @ A : (\tilde{s})\alpha \in \Gamma_i$ and $ch @ A : (\tilde{s})\overline{\alpha} \notin \Gamma_i$. (3) For term variables and variables we simply take the union.
3. (compatibility, 2) We write $\Delta_1 \asymp \Delta_2$ iff, whenever $\tilde{s}_i @ A_i : \alpha_i \in \Delta_i$, we have either: (1) $\{\tilde{s}_1\} \cap \{\tilde{s}_2\} = \emptyset$; (2) $\tilde{s}_1 = \tilde{s}_2$ and $\text{fsc}(\alpha_1) \cap \text{fsc}(\alpha_2) = \emptyset$; or (3) $\tilde{s}_1 = \tilde{s}_2$ and $\alpha_1 = \overline{\alpha_2}$.
4. (composition, 2) Suppose $\Delta_1 \asymp \Delta_2$. Then $\Delta_1 \odot \Delta_2$ is given as follows: (a) $\tilde{s} @ A : \alpha \in \Delta_1 \odot \Delta_2$ iff either (1) $\tilde{s} @ A : \alpha \in \Delta_1$ and $\{\tilde{s}\} \cap \text{fsc}(\Delta_2) = \emptyset$, or its symmetric case; (2) $\alpha = \alpha_1 \mid \alpha_2$, $\tilde{s}_1 @ A : \alpha_1 \in \Delta_1$ and $\tilde{s}_2 @ A : \alpha_2 \in \Delta_1$ such that $\text{fsc}(\alpha_1) \cap \text{fsc}(\alpha_2) = \emptyset$; and (b) $\tilde{s} : \perp$ iff $\tilde{s} @ A : \alpha \in \Delta_1$ and $\tilde{s} @ A : \overline{\alpha} \in \Delta_2$.

C Types for checking Well-Threadedness

We now introduce the type discipline which types all and only well-threaded interactions, via consistent global and local annotation. Given an interaction I , we want to check if it is well-threaded with respect to each thread compositionally. In order to do so, we define a new type system that selects all those interactions which are well-threaded. Let S, S', \dots range over vectors of session channels.

$$\Theta ::= \Theta \cdot \tau : (\uparrow, S) \mid \Theta \cdot \tau : (\downarrow, S) \mid \Theta, X : \Theta \mid \emptyset$$

Θ indicates, for each thread, associated session channels and the direction of the last action. The judgement has the form $\Theta \vdash \mathcal{A}$ meaning that annotation \mathcal{A} is well-threaded under the assumption about open threads in Θ (“open” in the sense that it has not been closed by initialisation). In Table 8 we report the typing rules where the function $\text{activeT}(\mathcal{A})$ takes the active thread of \mathcal{A} and the operation $\Theta_1 \odot \Theta_2$ gives an new environment containing all those $\tau \notin \text{dom}(\Theta_1) \cup \text{dom}(\Theta_2)$ and for all $\tau \in \text{dom}(\Theta_1) \cap \text{dom}(\Theta_2)$ we have that $\tau : \Theta_1(\tau) \cup \Theta_2(\tau)$ where, with an abuse of notation, we mean that they must have the same arrow and the union of the two session names sets. We use rule

Table 8 Typing Rules for checking Wellthreadedness

$$\begin{array}{c}
 \text{(WT-I)} \quad \frac{\Theta, \tau_1 : (\downarrow, S \uplus S'), \tau_2 : (\uparrow, S') \vdash \mathcal{A} \quad \text{activeT}(\mathcal{A}) = \tau_2 \quad S' \subseteq \{\bar{s}\}}{\Theta, \tau_1 : (\uparrow, S) \vdash A^{\tau_1} \rightarrow B^{\tau_2} : \text{ch}(\bar{s})(\nu \cdot) \mathcal{A}} \\
 \\
 \text{(WT-C)} \quad \frac{\Theta, \tau_1 : (\downarrow, S_1), \tau_2 : (\uparrow, S_2) \vdash \mathcal{A}_i \quad \text{activeT}(\mathcal{A}_i) = \tau_2 \quad J \neq \emptyset}{\Theta, \tau_1 : (\uparrow, S_1 \cup \{s\}), \tau_2 : (\downarrow, S_2 \cup \{s\}) \vdash \Sigma_i A^{\tau_i} \rightarrow B^{\tau_2} : s(\text{op}_i, e_i, x_i) \cdot \mathcal{A}_i} \\
 \\
 \text{(WT-A)} \quad \frac{\Theta \vdash \mathcal{A} \quad \text{activeT}(\mathcal{A}_i) = \tau}{\Theta \vdash x @ A^\tau := e \cdot \mathcal{A}} \quad \text{(WT-I T E)} \quad \frac{\Theta \vdash \mathcal{A}_i \quad \text{activeT}(\mathcal{A}_i) = \tau}{\Theta \vdash \text{if } e @ A^\tau \text{ then } \mathcal{A}_1 \text{ else } \mathcal{A}_2} \\
 \\
 \text{(WT-S)} \quad \frac{\Theta \vdash \mathcal{A}_i \quad \text{activeT}(\mathcal{A}_i) = \tau}{\Theta \vdash \mathcal{A}_1 + \mathcal{A}_2} \quad \text{(WT-Z)} \quad \frac{-}{\Theta, \tau : \epsilon \vdash \mathbf{0}} \\
 \\
 \text{(WT-R)} \quad \frac{\Theta, \tau : (\downarrow, s) \vdash \mathcal{A} \quad \text{activeT}(\mathcal{A}) = \tau}{\Theta, \tau : (\downarrow, \{\}) \vdash (\nu s)^\tau \mathcal{A}} \quad \text{(WT-V)} \quad \frac{-}{\Theta, X : \Theta \vdash X^\tau} \\
 \\
 \text{(WT-P)} \quad \frac{\Theta_i \vdash \mathcal{A}_i \quad \text{activeT}(\mathcal{A}_i) = \tau}{\Theta_1 \odot \Theta_2 \vdash \mathcal{A}_1 \upharpoonright^\tau \mathcal{A}_2} \quad \text{(WT-R)} \quad \frac{\Theta, X : \Theta \vdash \mathcal{A} \quad \text{activeT}(\mathcal{A}) = \tau}{\Theta \vdash \mu^\tau X^\tau \cdot \mathcal{A}} \\
 \\
 \text{(WT-P E)} \quad \frac{\Theta \vdash \mathcal{E}_i}{\Theta \vdash \mathcal{E}_1 \upharpoonright^\tau \mathcal{E}_2}
 \end{array}$$

(WT-P E) for treating top-level parallel composition nodes (denoted by \mathcal{E}_i). This corresponds to the requirement **(G3)** in the definition of consistent annotation.

We can now state the following theorem.

Theorem 25.

1. An annotated interaction \mathcal{A} is consistent if and only if $\Theta \vdash \mathcal{A}$.
2. Let \mathcal{A} be an annotation of I and let $\Theta \vdash \mathcal{A}$. If $(\sigma, I) \rightarrow (\sigma', I')$ then there exists an annotation \mathcal{A}' of I' such that $\Theta \vdash \mathcal{A}'$.

The typing system above induces a simple algorithm which can inductively annotate interactions with threads and, along the way, checks its well-threadedness. For details we refer the reader to [11].

C.1 Definitions for thread projections

This subsection defines $\tau_1 \equiv_{\mathcal{A}} \tau_2$ used in the end point projection. We first define the mapping $\text{threads}(\mathcal{A}, \text{ch})$ in Table 9. If two input threads are for the same service channel, then they are equivalent. Below $\text{channels}(\mathcal{A})$ indicates the set of service channels occurring in \mathcal{A} .

Table 9 Inductive definition of the function `threads`

$$\text{threads}(A^{\tau_1} \rightarrow B^{\tau_2} : ch(\nu s). \mathcal{A}', ch) \stackrel{\text{def}}{=} \begin{cases} \{\tau_2\} \cup \text{threads}(\mathcal{A}', ch) & \text{if } ch = ch' \\ \text{threads}(\mathcal{A}', ch) & \text{otherwise} \end{cases}$$

$$\text{threads}(\mathcal{A}, ch) \stackrel{\text{def}}{=} \text{threads}(\mathcal{A}', ch) \quad \text{if } \mathcal{A} \in \{A^{\tau_1} \rightarrow B^{\tau_2} : s(\text{op}, x, \cdot)\mathcal{A}', x @ A^\tau := e. \mathcal{A}', \mu X_r^A . \mathcal{A}'\}$$

$$\text{threads}(\mathcal{A}, ch) \stackrel{\text{def}}{=} \text{threads}(\mathcal{A}'_1, ch) \cup \text{threads}(\mathcal{A}'_2, ch) \quad \text{if } \mathcal{A} \in \{\text{if } e @ A^\tau \text{ then } \mathcal{A}'_1 \text{ else } \mathcal{A}'_2, \mathcal{A}'_1 + \mathcal{A}'_2, \mathcal{A}'_1 \mid \mathcal{A}'_2\}$$

$$\text{threads}(X_r^A, ch) \stackrel{\text{def}}{=} \text{threads}(\mathbf{0}, ch) \stackrel{\text{def}}{=} \emptyset$$

Definition 26. Given a well-threaded annotated interaction \mathcal{A} , for all $\tau \in \mathcal{A}$, we define the equivalence class $[\tau]^{\mathcal{A}} \subseteq \mathbb{N}$ as

$$[\tau]^{\mathcal{A}} = \begin{cases} \text{threads}(\mathcal{A}, ch) & \text{if } \exists ch \in \text{channels}(\mathcal{A}) \text{ s.t.} \\ & \tau \in \text{threads}(\mathcal{A}, ch) \\ \{\tau\} & \text{otherwise.} \end{cases}$$

Moreover, given τ_1 and τ_2 in \mathcal{A} , we write $\tau_1 \equiv_{\mathcal{A}} \tau_2$ whenever there exists $\tau \in \mathcal{A}$ such that $\tau_1, \tau_2 \in [\tau]^{\mathcal{A}}$.