Problems Solved:

31	32	33	34	35

Name:

Matrikel-Nr.:

Problem 31. For a Turing machine M let $P(M)$ be the following property: *If M runs at least 1000 steps on a word w, then $w \in L(M)$.* Note that there is no statement about acceptance or non-acceptance if the machine runs less than 1000 steps.
In the following let M be a Turing machine that has the property $P(M)$.

1. Is there a Turing machine E with $P(E)$ such that $\varepsilon \in L(E)$.
2. Is there a Turing machine E with $P(E)$ such that $\varepsilon \notin L(E)$.
3. Is the property of $L(M)$ to contain the empty word, decidable?
4. Is $L(M)$ recursively enumerable?
5. Is the complement $\overline{L(M)}$ recursively enumerable?
6. Is $L(M)$ recursive?
7. Is $L(M)$ necessarily finite?
8. Is $L(M)$ necessarily infinite?

Justify your answers.

Problem 32. Which of the following problems are decidable? In each problem below, the input of the problem is the code $\langle M \rangle$ of a Turing machine M with input alphabet $\{0, 1\}$.

(a) Does M have at least 4 states?

(b) Is $L(M) \subseteq \{0, 1\}^*$?

(c) Is $L(M)$ recursive?

(d) Is $L(M)$ finite?

(e) Is $10101 \in L(M)$?

(f) Is $L(M)$ not recursively enumerable?

(g) Does there exist a word $w \in L(M)$ such that M does not halt on w?

Justify your answer.

Problem 33. Let $M_0, M_1, M_2, \ldots$ be a list of all Turing machines with alphabet $\Sigma = \{0, 1\}$ such that the function $i \mapsto \langle M_i \rangle$ is computable. Let $w_i := 10^i10^i1$ for all natural numbers i. Let $A := \{w_i \mid i \in \mathbb{N} \wedge w_i \in L(M_i)\}$ and $\overline{A} = \Sigma^* \setminus A$.

(a) Is $\overline{A}$ recursively enumerable? (Justify your answer.)

(b) Suppose there is an oracle X_{Delphi} that decides the Halting problem, i. e., you can give to X_{Delphi} the code $\langle M \rangle$ of a a Turing machine M and a word w and X_{Delphi} returns 1 (YES) or 0 (NO) depending on whether or not M halts on w.

Show that one can construct an Oracle-Turing machine T (which is allowed by a special extension to give some word $\langle M \rangle$ (a Turing machine code) and a word w to X_{Delphi} and gets back 1 or 0 depending on whether or not M halts on w) such that $L(T) = \overline{A}$.

(c) Does it follow from (a) and (b) that X_{Delphi} is not a Turing machine? Justify your answer. Note that you are not allowed to use the fact that the Halting problem is undecidable, but you must give a proof that only follows from (a) and (b).

Problem 34. Show that the Acceptance Problem is reducible to the restricted Halting problem. First explain clearly which Turing machine you have to construct to prove this statement and then give a reasonably detailed description of this construction.

Problem 35. Let a language $L = L(T) \subseteq \{0,1\}^*$ be given by the code of a Turing machine $\langle T \rangle$. It is known that $\varepsilon \in L$.
Let S_0 be the set of Turing machines of the form $(Q, \{0, 1, X, \sqcup\}, \sqcup, \{0, 1\}, \delta, q_0, \emptyset)$.
Let S_1 be the set of Turing machines of the form $(Q, \{0, 1, X, \sqcup\}, \sqcup, \{0, 1\}, \delta, q_0, Q)$.
Is it decidable whether $L = L(M)$ and $M \in S_0$? That is: Is there a Turing machine D_0 such that it takes a word w as input and returns "yes" if $w = \langle M \rangle$ for a TM $M \in S_0$ with the property $L(M) = L$, and returns "no" otherwise?
What is the answer, if you replace S_0 by S_1? Justify your answers.