Puiseux Series Solutions of Autonomous AODEs

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In particular, existence, uniqueness and convergence of the solutions can be ensured. Moreover, the solutions are in the field extension $\mathbb{Q}[s, c]$, where s is algebraic and c transcendental over $\mathbb{Q}$.

Algorithmic Aspects

Computations highly rely on local parametrizations

D. Duval, *Rational Puiseux expansion*. Compositio Mathematica, 70:119–154, 1989.

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For systems of AODEs additionally differential pseudo-remainder computations are necessary.

J.F. Ritt *Differential Algebra*. American Mathematical Society, 1950.