Problems Solved:

21	22	23	24	25

Name:

Matrikel-Nr.:

Problem 21. Let $\Sigma = \{a, b\}$. We encode a and b on the input tape of a RAM by 1 and 2 and a word $w \in \Sigma^*$ by a respective sequence of 1's and 2's.
We say that a RAM R *accepts* a word $w \in \Sigma^*$ if R starts with the coded word w on its input tape and terminates after having written a non-zero number on its output tape. We define $L(R) := \{w \in \Sigma^* \mid R \text{ accepts } w\}$.
Let F be a RAM that terminates for every input and whose program does not contain "loops", i.e., each instruction is executed at most once.
Derive answers for the following questions. (Give ample justifications, just saying 'yes' or 'no' is not enough.)

1. Is $L(F)$ as a language over Σ finite?
2. Is $L(F)$ as a language over Σ regular?

Problem 22. Write a RAM program that from a given natural number n prints its binary representation. In order to simplify the problem the output shall be in low positions first format, i.e., the number 8_{10} is 0001_2 but not 1000_2.
Hint: please note that the computation of the quotient respectively remainder of a division by 2 can be implemented by the repeated subtraction of 2.

Problem 23. In the following use *only* the definition of a *loop program* as given in Def. 23 of the lecture notes, Section 3.2.2. Note that it is not allowed to use abbreviations like

```
xi := xj - xk;
xi := xj + xk;
```

Furthermore, the variables in a loop program are only $x_0, x_1, \ldots$

1. Show that the function

$$s(x_1, x_2) = \begin{cases} 1 & \text{if } x_1 < x_2, \\ 0 & \text{otherwise} \end{cases}$$

   is loop computable. I.e. give an explicit loop program for s.

2. Write a loop program that computes the function $d : \mathbb{N}^2 \to \mathbb{N}$ where $d(x_1, x_2)$ is $k \in \mathbb{N}$ such that $k \cdot (x_2 + 1) = x_1 + 1$ if such a k exists. The result is $d(x_1, x_2) = 0$, if a k with the above property does not exist.

   For simplicity in the program for d, you are allowed to use a construction like the following (with the obvious semantics) where P is an arbitrary loop program.

   IF xi < xj **THEN** P **END**;

   Note: Only $<$ is allowed in the condition and there must be a variable before and a variable after the $<$ sign and $1 < x_j$ is not allowed. Note also that there is no "ELSE" branch.

Problem 24. Provide a loop program that computes the function $f(n) = \sum_{k=1}^{n} k(k+1)$, and thus show that f is loop computable.
You are only allowed to use the constructs given in Definition 23 of the lecture notes.

Problem 25. Suppose P is a while-program that does not contain any `WHILE` statements, but might modify the values of the variables x_1 and x_2.
Transform the following program into Kleene's normal form.
Hint: first translate the program into a goto program, replace the `GOTO`s by assignments to a control variable, and add the `WHILE` wrapper.

```
x0 := 0
WHILE x1 DO
    x1 := x1 - 1;
    x2 := x1;
    WHILE x2 DO
        P;
    END;
END;
x0 := x0 + 1
```