Problems Solved:

11	12	13	14	15

Name:

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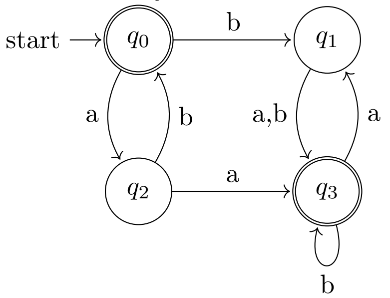
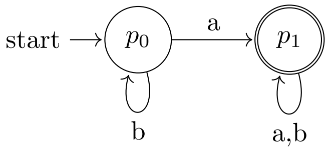
DEAIntersect-01

Problem 11. Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be two DFSM over the alphabet Σ. Let $L(M_1)$ and $L(M_2)$ be the languages accepted by M_1 and M_2, respectively.
Construct a DFSM $M = (Q, \Sigma, \delta, q, F)$ whose language $L(M)$ is the intersection of $L(M_1)$ and $L(M_2)$. Write down Q, δ, q, and F explicitly.
Hint: M simulates the parallel execution of M_1 and M_2. For that to work, M "remembers" in its state the state of M_1 as well as the state of M_2. This can be achieved by defining $Q = Q_1 \times Q_2$.
Demonstrate your construction with the following DFSMs.



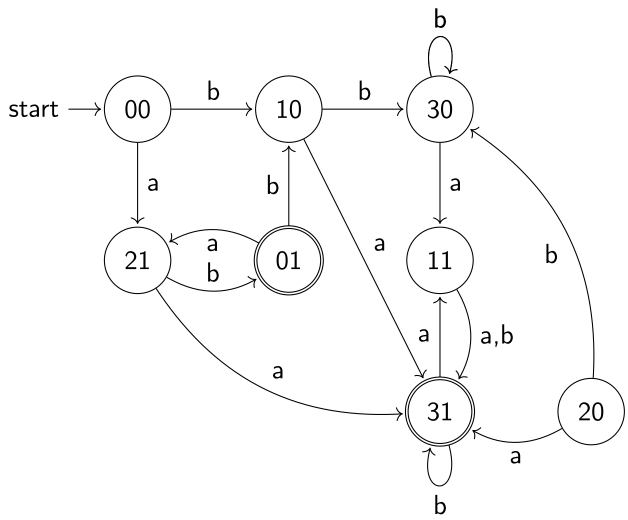


Solution of Problem 11:

- $Q = Q_1 \times Q_2$,
- $q = (q_1, q_2)$, (initial state)
- $\delta((p, q), c) = (\delta(p, c), \delta(q, c))$ for all $p \in Q_1$ and $q \in Q_2$ and $c \in \Sigma$.
- $F = (F_1 \times Q) \cap (Q \times F_2) = F1 \times F2$.

For the concrete example, we have

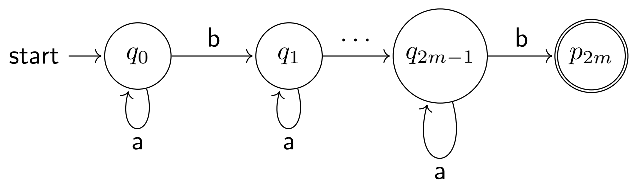
- $Q = Q_1 \times Q_2$. We abbreviate by $Q = \{00, 01, 10, 11, 20, 21, 30, 31\}$.
- $q = 00$, (initial state)
- $F = \{01, 31\}$



Pumping2020 **Problem 12.** Let $m > 1$ be a natural number and $r = (a^*b)^m$ be a regular expression.
Definition: If a is a regular expression and n is a natural number, then $a^n = a \cdots a$ is the regular expression that results from the n-fold concatenation of a. For example: $a^3 = aaa$.
Let L_1 and L_2 be the languages defined as follows: $L_1 := L(r)$, $L_2 := \{(a^k b)^m \mid k \in \mathbb{N}\}$.
Are L_1 and L_2 regular languages? Provide solid arguments to your answers.

Solution of Problem 12:

L_1 is regular. In fact, for any concrete value of m there is a concrete regular language. The following DFSM may be taken as a pattern:



Let m be a concrete natural number, such that $m > 1$. Suppose L_2 is regular.

Then there is a DFSM $A = (Q, \{a, b\}, q_0, F, \delta)$ with $N := |Q|$, such that it accepts L_2.

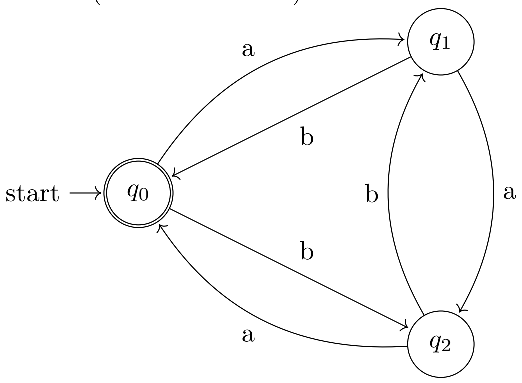
Let $z = a^N b(a^N b)^{m-1}$. Then we have that $z \in L_2$.

According to the Pumping Lemma we can find $u, v, w \in \{a, b\}^*$ with $z = uvw \in L_2$, $|uv| \leq N$, $|v| \geq 1$ such that $uv^i w \in L_2$ for any $i \geq 0$.

Then u and v must have the form $u = a^k$, $v = a^l$ for some k, l with $k+l \leq N$ and $l > 0$. Then, it follows that $a^{N+i*l}b(a^N b)^{m-1}$ also belongs to L_2, which is not the case. Therefore, the assumption is wrong and L_2 is not regular.

Arden-01 **Problem 13.** Let M_1 be the DFSM with states $\{q_0, q_1, q_2\}$ whose transition graph is given below. Determine a regular expression r such that $L(r) = L(M_1)$.

Show the *derivation* of the the final result by the technique based on Arden's Lemma (see lecture notes).



Solution of Problem 13:

Analogous to the explanation following Arden's lemma in the lecture notes, we derive the following system.

$$X_0 = aX_1 + bX_2 + \varepsilon$$
$$X_1 = bX_0 + aX_2$$
$$X_2 = aX_0 + bX_1$$

Now plug equation (3) in (2):

$$\begin{aligned} X_1 &= bX_0 + a(aX_0 + bX_1) = \\ &= abX_1 + (b + aa)X_0 = \\ &= (ab)^*(b + aa)X_0 \end{aligned}$$

$$\begin{aligned} X_0 &= a(ab)^*(b + aa)X_0 + baX_0 + bb(ab)^*(b + aa)X_0 + \varepsilon \\ &= ((a + bb)(ab)^*(b + aa) + ba)X_0 + \varepsilon \\ &= ((a + bb)(ab)^*(b + aa) + ba)^* \end{aligned}$$

Fianally we obtain that $r = ((a + bb)(ab)^*(b + aa) + ba)^*$.

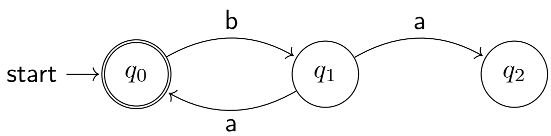
H **Problem 14.** Let r be the following regular expression.

$$a \cdot a \cdot (b \cdot a)^* \cdot b \cdot b^*$$

Construct a nondeterministic finite state machine N such that $L(N) = L(r)$. Show the derivation of the result by following the technique presented in the proof of the theorem *Equivalence of Regular Expressions and Automata* (see lecture notes).

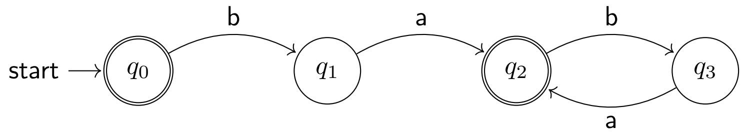
Solution of Problem 14:

A NFSM for $(ba)^*$ is by case $r = r_1^*$

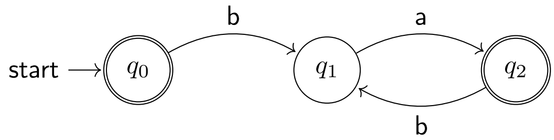


From state q_2 the final state cannot be reached, thus, we can remove it.

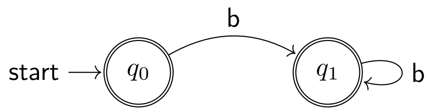
The construction of the proof relies on automata that have no transition back to the initial state. Thus we "normalize" the above graph (as described in the beginning of the proof) to



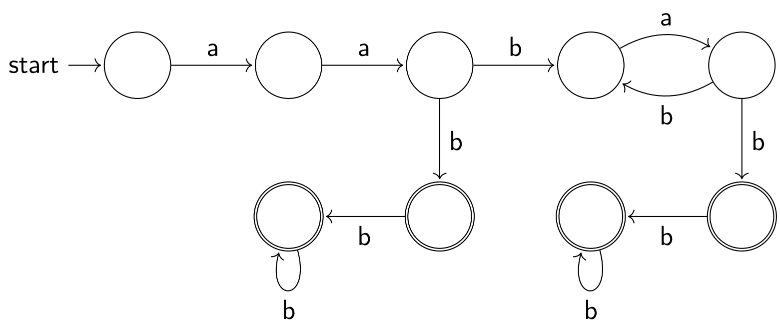
and simplify to the following graph.



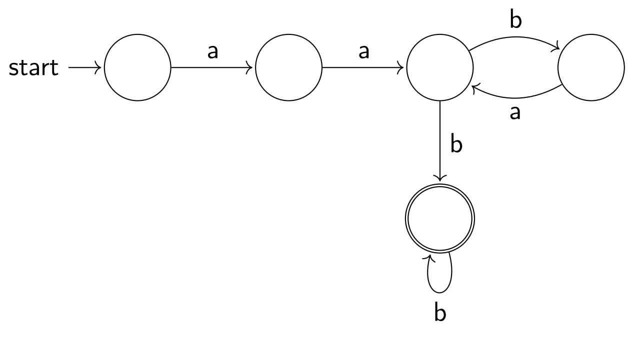
Similarly, for b^* we get:



Putting everything together yields



The above automaton can be simplified to



which defines the required NFSM N.

`InversePumping` **Problem 15.** Show that the language $L = \{a^m b^n \mid m, n \in \mathbb{N} \land m \geq 2n\}$ is not regular.

Solution of Problem 15:

Suppose L is regular.

Then there is a DFSM $A = (Q, \{a, b\}, q_0, F, \delta)$ with $N := |Q|$, such that it accepts L.

Let $z = a^{2N}b^N$. Then we have that $z \in L$.

According to the Pumping Lemma we can find $u, v, w \in \{a, b\}^*$ with $z = uvw \in L$, $|uv| \leq N$, $|v| \geq 1$ such that $uv^iw \in L$ for any $i \geq 0$.

We now have $u = a^m$, $v = a^{k-m}$, $w = a^{2N-k}b^N$ for certain natural numbers m and n such that $0 \leq m < k \leq N$.

Then, it follows that $uv^0w = a^{2N-(k-m)}b^N \in L$, which is not the case. Therefore, the assumption is wrong and L is not regular.