Problems Solved:

16	17	18	19	20

Name:

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Problem 16. Construct a Turing machine $M = (Q, \Gamma, \sqcup, \{0,1\}, \delta, q_0, F)$ such that $L(M) = \left\{1^k 0 1^{k+1} \,\middle|\, k \in \mathbb{N}\right\}$. Write down Q, Γ, F and δ explicitly.

Problem 17. Give an informal description of a Turing machine M which for any two natural numbers computes their least common multiple. You may assume that the numbers are represented in their unary form, e.g., the pair of numbers 5 and 3 is represented as 111110111.
Hint: You may use a Turing machine with two tapes, as described in the lecture notes.

Problem 18. We give the Turing machine $M = (Q, \Sigma, \Gamma, q_0, F, \delta)$ with $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$, $F = \{q_3\}$ and the transition function

$$\delta \;:\; Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$

with $\delta(q_0, 1) = (q_0, 1, R)$, $\delta(q_0, 0) = \delta(q_1, 1) = (q_1, 1, R)$, $\delta(q_1, \sqcup) = (q_2, \sqcup, L)$, $\delta(q_2, 1) = (q_3, \sqcup, R)$. For any other values δ is not defined.
Write down the sequence of configurations of M starting on the word 110111.

Problem 19. Using the Turing Machine Simulator
`http://math.hws.edu/eck/js/turing-machine/TM.html`
construct a Turing machine M over $\Sigma = \{0\}$ which computes the "Double n" function $d : \mathbb{N} \to \mathbb{N}$ given by $d(n) = 2n$.
Use unary representation: A number n is represented by the string 0^n consisting of n copies of the symbol 0. As an extension of the Turing Machine concept, an infinite tape in both directions may be used. The solution shall be the JSON expression generated for your machine. For example, the JSON expression generated for the "Binary Increment" TM, copied from the above website is:

```
{
   "name": "Binary Increment",
   "max_state": 25,
   "symbols": "xyzabc01$@",
   "tape": "10011111",
   "position": 7,
   "rules": [
      [ 0, "#", "1", 1, "R" ],
      [ 0, "0", "1", 1, "R" ],
      [ 0, "1", "0", 0, "L" ],
      [ 1, "#", "#", "h", "L" ],
      [ 1, "0", "0", 1, "R" ],
      [ 1, "1", "1", 1, "R" ]
   ]
}
```

Problem 20. Write down explicitly an enumerator G such that $\mathrm{Gen}(G) = \left\{0^{2n} \,\middle|\, n \in \mathbb{N}\right\}$.
Since in the lecture notes it has not been *formally* defined, how a Turing machine with two tapes works, you may describe the transition function as

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, L\} \times (\Gamma \cup \{\boxtimes\})$$

in the following way: If G is in state q and reads the symbol c from the working tape, and

$$\delta(q, c) = (q', c', d, c'')$$

then G goes to state q', replaces c by c' on the working tape and moves the working tape head in direction d. Moreover, unless $c'' = \boxtimes$, the symbol c'' is written on the output tape and the output tape head is moved one position forward. If, however, $c'' = \boxtimes$, nothing is written on the output tape and the output tape head rests in place.
Hint: There exists a solution with only 4 states.