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Induction upon $n \in \mathbb{N}^+$

Base case:

Check if the statement holds for $n=1$.

Indeed $\sum_{i=1}^{1} i = \frac{1 \cdot (1+1)}{2}$

Induction hypothesis:

Assm. that for one concrete $k$

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Show that the statement

holds for $n = k+1$

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We need to show that

$$\sum_{i=1}^{k+1} i = \frac{(k+1) \cdot (k+1+1)}{2}$$

Indeed:

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$$= \frac{k\cdot(k+1)}{2} + (k+1) =$$

$$= \frac{k\cdot(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k(k+1)+2(k+1)}{2}$$

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$$= \frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+1+1)}{2}$$

We conclude that the statement holds for any $n \in \mathbb{N}^+$.