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Since $\frac{a}{b} = \sqrt{2}$, we obtain

$$\frac{a^2}{b^2} = 2 .$$

This leads to

$$a^2 = 2 \cdot b^2 .$$

It follows that

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Since $\underline{a}$ is an even number,

we have $a = 2p$ for some p.

We obtain

$$(2p)^2 = 2b^2$$

$$4p^2 = 2b^2$$

$$2p^2 = b^2$$

From here it follows that

b^2 is an even number and so

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From this we conclude that our assumption that there are a, b s.t. $\sqrt{2} = \frac{a}{b}$ is impossible.

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From thes we conclude that our assumption that there are a, b s.t. $\sqrt{2} = \frac{a}{b}$ is impossible.

Therefore $\sqrt{2} \notin \mathbb{Q}$.