Problems Solved:

1	2	3	4	5

Name:

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Problem 1. Show by induction that

$$\sum_{i=1}^{n}(3i-2) = \frac{n(3n-1)}{2}$$

for $n \geq 1$.

Problem 2. Let $L \subseteq \mathbb{N} \times \mathbb{N}$ be the smallest set of pairs obeying the following:

- $\langle 0, 0 \rangle \in L$;
- if $\langle m, n \rangle \in L$ for some m and n, then $\langle m+5, n+1 \rangle \in L$ as well.

Show by induction that if $\langle m, n \rangle \in L$ then $m + n$ is a multiple of 3.
Hint: Recall that a natural number i is a multiple of 3 if there is some natural number j such that $i = 3 \cdot j$.

Problem 3. Show $\sqrt{6} \notin \mathbb{Q}$ by an indirect proof.
Hint: `http://en.wikipedia.org/wiki/Square_root_of_2#Proofs_of_irrationality`.
Hint: Note that the fact that $\sqrt{2}$ and $\sqrt{3}$ are irrational does not imply that $\sqrt{2} \cdot \sqrt{3}$ is irrational as well. For example $\sqrt{2} \cdot \sqrt{8}$ is rational although both $\sqrt{2}$ and $\sqrt{8}$ are irrational.

Problem 4. Construct a nondeterministic finite state machine M over the alphabet $\{a, b, l, -\}$ such that it accepts the language $L(M) = \{bla\}$.

(a) Give the formal definition of M and draw the graph.

(b) What has to be changed in order for the machine to accept all finite strings of the form bla, $bla - bla$, $bla - bla - bla \ldots$? (The empty word shall not be accepted.)

Problem 5. Construct nondeterministic finite state machines for:

(a) the language L_1 of all strings over $\{0, 1\}$ that contain 010 as a substring.

(b) the language L_2 of all strings over $\{0, 1\}$ that contain the letters 0, 1, 0 in exactly that order. (Note that before, in between and after these three letters any number of other letters may occur).

Give the formal definitions of the machines and draw their graphs.
Your two machines must not use more than 4 states. Moreover, they should only differ in their transition functions.