31	32	33	34	35

Problems Solved:

Name:

Matrikel-Nr.:

Problem 31. Let $M = (Q, \Gamma, \sqcup, \Sigma, \delta, q_1, F)$ be a Turing maching with $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$, $Q = \{q_1, \ldots, q_n\}$, $F = \{q_2\}$, and transition function δ. We denote the symbols $0, 1, \sqcup$ in this order by X_1, X_2, X_3 and the head movement direction L, R by D_1, D_2. An operation $\delta(q_i, X_j) = (q_k, X_l, D_m)$ shall be coded as $0^i10^j10^k10^l10^m$. The Turing machine M itself shall be coded as

$$111code_1 11code_2 \ldots 11code_r 111$$

where each $code_1$ up to $code_r$ encode the operations given by δ. We denote such a code of a Turing machine M by $[M]$. Note that this encoding is different from the code $\langle M \rangle$ that is given in the lecture notes.

1. Let $w \in \Sigma^*$. Is it decidable whether w is the code of a Turing machine?
2. Is there a Turing machine U_ε that takes an arbitrary word $w \in \Sigma^*$ as input, checks whether this word is the code of a Turing machine and, if yes, simulates the behaviour of the respective Turing machine when started on the empty word?
3. Is it decidable whether there is a Turing machine U_ε as described in the previous question?
4. If the answer to the second question is “yes”, is it decidable whether such a Turing machine U_ε halts on every word $w \in \Sigma^*$ that is the code of a Turing machine.
5. Optional: If the answer to the second question is “yes”, is it decidable whether a Turing machine U_ε with the property given in Question 2 halts on every word $w \in \Sigma^*$ that is **not** the code of a Turing machine.

Give reasonable arguments for your answers. Informal reasoning is enough, but simply stating “yes” or “no” does not count as a solution of this exercise.

Problem 32. Let

$$L = \left\{ w \in \{0,1\}^* \,\middle|\, \exists M : M \text{ is a TM} \wedge \langle M \rangle = w \wedge L(M) \text{ is recursive} \right\}.$$

Is L recursive?

Problem 33. Let L be a language over the alphabet $\Sigma = \{0, 1\}$ that is generated by some Turing machine N. For which L is the following problem semi-decidable? For which L is it decidable?
Input of the problem (*instance* of the problem): the code $\langle M \rangle$ of a Turing machine M.
Question of the problem: $L(M) \cap L \neq \emptyset$?

Problem 34. Which of the following problems are decidable? Justify your answers. In each problem below, the input of the problem is the code $\langle M \rangle$ of a Turing machine M with input alphabet $\{0, 1\}$.

1. Is $L(M)$ empty?
2. Is $L(M)$ finite?
3. Is $L(M)$ regular?
4. Is $L(M) \subseteq \{0,1\}^*$?
5. Is $L(M)$ not recursively enumerable?
6. Does M have an even number of states?

Problem 35. Show that the Acceptance Problem is reducible to the restricted Halting problem. First explain clearly which Turing machine you have to construct to prove this statement and then give a reasonably detailed description of this construction.