Gruppe	Popov (8:30)	Popov (9:15)	Popov (10:15)	Hemmecke (10:15)	Hemmecke (11:00)
Name			Matrikel		SKZ

Klausur 1

Berechenbarkeit und Komplexität

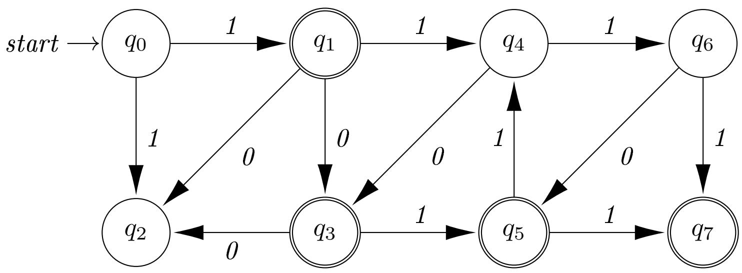
22. November 2019

Part 1 NFSM2019

Let N be the nondeterministic finite state machine

$$(\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, \{0, 1\}, \nu, \{q_0\}, \{q_1, q_3, q_5, q_7\}),$$

whose transition function ν is given below.



1		no

Is $1001100111 \in L(N)$*?*

The sequence is not defined by the transition.

2	yes	

Is $101111 \in L(N)$*?*

Follow the states $q_0, q_1, q_3, q_5, q_4, q_6, q_5$.

3		no

Is $L(N)$ *finite?*

4	yes	

Does there exist a regular expression r such that $L(r) = \overline{L(N)} = \{0,1\}^* \setminus L(N)$*?*

$L(N)$ is regular and so is its complement.

5	yes	

Is $\overline{L(N)}$ *recursively enumerable?*

$L(N)$ is regular. Hence, $\overline{L(N)}$ is regular, and thus also recursively enumerable.

6	yes	

Is there a deterministic finite state machine M with less than 2019 states such that $L(M) = L(N)$*?*

According to the subset construction, there must be a DFSM with at most $2^8 = 256$ states.

7	yes	

Is there an enumerator Turing machine G such that $Gen(G) = L(N)$*?*

8	yes	

Does there exists a deterministic finite state machine D such that $L(D) = L(N) \circ \overline{L(N)}$*?*

$L(N)$ and $\overline{L(N)}$ are both regular. Concatenation of two regular languages gives a regular language.

Part 2 Computable2019

Let M be a Turing machine such that whenever M accepts a word, it does so in no more than 2019 steps.

9	yes		*Is $L(M)$ recursively enumerable?*
10	yes		*Is $L(M)$ recursive?*

Start M with input w and execute 2019 steps. If w has been accepted then $w \in L(M)$, otherwise $w \notin L(M)$. Therefore, $L(M)$ and $\overline{L(M)}$ are both recursively enumerable.

11		no	*Let L be a recursively enumerable language. Can it be concluded that $L(M) \cap L$ is recursive?*

Intersection of recursive and recursively enumerable languages is recursively enumerable but not necessarily recursive.

12	yes		*Let L be a recursively enumerable language. Can it be concluded that $\overline{L}$ is recursive, provided that $\overline{L}$ is recursively enumerable?*

If a language L and its complement $\overline{L}$ are both recursively enumerable, then by Theoem 8 (Skriptum) L is recursive.

13	yes		*Does there exist a Turing-computable function that is not LOOP-computable?*
14		no	*Is every total WHILE-computable function also LOOP-computable?*
15	yes		*Let f be a LOOP-computable function and $g : \{\sharp\}^* \to \{\sharp\}^*$ be defined by $g(\sharp^n) = \sharp^{f(n)}$ for all $n \in \mathbb{N}$. Is g Turing-computable?*

Part 3 *Pumping2019*

Let

$$L_1 = \{\, a^m b^n \mid m, n \in \mathbb{N}, m \le n \,\},$$
$$L_2 = \{\, a^n b^n \mid n \in \mathbb{N}, n < 2019 \,\}.$$

16		no	*Is there a regular expression r such that $L(r) = L_1$?*
17	yes		*Is there a deterministic finite state machine M such that $L(M) = \overline{L_2} := \{a,b\}^* \setminus L_2$?*

L_2 is regular, i.e., its complement $\overline{L_2}$ is also regular.

18	yes		*Is there an enumerator Turing machine G such that $Gen(G) = L_1$?*
19	yes		*Is there a Turing machine M such that $L(M) = L_1 \cup L_2$?*
20	yes		*Is there a deterministic finite state machine D such that $L(D) = L_1 \cap L_2$?*

The language $L_1 \cap L_2$ is finite and thus regular.

Part 4 *WhileLoop2019*

Let a function $f : \mathbb{N}^3 \to \mathbb{N}$ be defined by

$$f(x,y,z) := \begin{cases} y & \text{if } x = y, \\ z & \text{if } x < y, \\ 0 & \text{otherwise.} \end{cases}$$

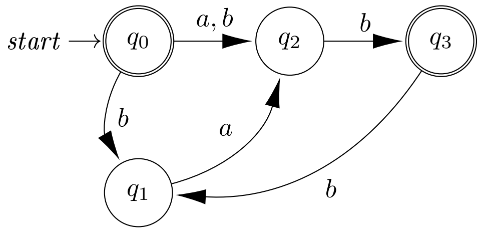
Let f' be defined like f, but with the exception that f' is undefined if one of the arguments is equal to 2019.

21	yes		*Is f a LOOP computable function?*
22		no	*Is f' a LOOP computable function?*
23	yes		*Is f' a WHILE computable function?*

Part 5 Open2019

((2 points))

Let $N = (Q, \Sigma, \delta, q_0, F)$ be a nondeterministic finite state machine with $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, $S = \{q_0\}$, $F = \{q_0, q_3\}$, and transition function δ as given below.



1. *Let X_i denote the regular expression for the language accepted by N when starting in state q_i.*

   Write down an equation system for $X_0, \ldots, X_3$.

2. *Give a regular expression r such that $L(r) = L(N)$ (you may apply Arden's Lemma to the result of 1).*

$$
\begin{aligned}
X_0 &= bX_1 + (a + b)X_2 + \varepsilon \\
X_1 &= aX_2 \\
X_2 &= bX_3 \\
X_3 &= bX_1 + \varepsilon \\
r &= ((a + b) + ba)(bba)^*b + \varepsilon
\end{aligned}
$$

or alternatively:

$$r = ((a + b) + ba)b(bab)^* + \varepsilon$$