

Problems Solved:

21	22	23	24	25
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Problem 21. Let $\Sigma = \{a, b\}$. We encode a and b on the input tape of a RAM by 1 and 2 and a word $w \in \Sigma^*$ by a respective sequence of 1's and 2's.

We say that a RAM R *accepts* a word $w \in \Sigma^*$ if R starts with the coded word w on its input tape and terminates after having written a non-zero number on its output tape. We define $L(R) := \{w \in \Sigma^* \mid R \text{ accepts } w\}$.

Let F be a RAM that terminates for every input and whose program does not contain "loops", i.e., each instruction is executed at most once.

Derive answers for the following questions. (Give ample justifications, just saying 'yes' or 'no' is not enough.)

1. Is $L(F)$ as a language over Σ finite?
2. Is $L(F)$ as a language over Σ regular?

Problem 22. Write a RAM program that computes the function $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(n) = \sum_{k=1}^n k$ such that the program reads an integer n from the input tape and (upon termination) has written $f(n)$ to its output tape.

Note that it is not specified what the RAM should do if it reads a negative integer. Your RAM program can thus assume that the input is non-negative.

Problem 23. In the following use *only* the definition of a *loop program* as given in Def. 23 of the lecture notes, Section 3.2.2. Note that it is not allowed to use abbreviations like

$x_i := x_j - x_k;$
 $x_i := x_j + x_k;$

Furthermore, the variables in a loop program are only x_0, x_1, \dots

1. Show that the function

$$s(x_1, x_2) = \begin{cases} 1 & \text{if } x_1 < x_2, \\ 0 & \text{otherwise} \end{cases}$$

is loop computable. I.e. give an explicit loop program for s .

2. Write a loop program that computes the function $d : \mathbb{N}^2 \rightarrow \mathbb{N}$ where $d(x_1, x_2)$ is $k \in \mathbb{N}$ such that $k \cdot (x_2 + 1) = x_1 + 1$ if such a k exists. The result is $d(x_1, x_2) = 0$, if a k with the above property does not exist.

For simplicity in the program for d , you are allowed to use a construction like the following (with the obvious semantics) where P is an arbitrary loop program.

IF $x_i < x_j$ **THEN** P **END**;

Note: Only $<$ is allowed in the condition and there is no "ELSE" branch.

Problem 24. Provide a loop program that computes the function $f(n) = n! = \prod_{k=1}^n k$, and thus show that f is loop computable. In your program you are only allowed to use statements from Definition 23 of the lecture notes.

Problem 25. Suppose P is a while-program that does not contain any WHILE statements, but might modify the values of the variables x_1 and x_2 . Transform the following program into Kleene's normal form.

Hint: first translate the program into a goto program, replace the GOTOs by assignments to a control variable, and add the WHILE wrapper.

```
x0 := 0
WHILE x1 DO
  x1 := x1 - 1;
  x2 := x1;
  WHILE x2 DO
    P;
  END;
END;
x0 := x0 + 1
```