**Problems Solved:**

| 16 | 17 | 18 | 19 | 20 |
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**Name:**

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**Problem 16.** Given the language $L := \left\{ aa^{-1} \,\middle|\, a \in \Sigma^* \right\}$ where $\Sigma = \{0, 1\}$. Give an informal description of a Turing machine $M$, s.t., $L = L(M)$. You may use the following definition: $(a_1 a_2 \cdots a_k)^{-1} := a_k a_{k-1} \ldots a_1$ for $a_1, a_2, \ldots, a_k \in \Sigma$.

**Problem 17.** Given the Turing machine $M = (Q, \Sigma, \Gamma, q_0, F, \delta)$ with $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \sqcup\}$, $F = \{q_3\}$ and the transition function

$$\delta \;:\; Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$

with $\delta(q_0, 1) = (q_0, 1, R)$, $\delta(q_0, 0) = \delta(q_1, 1) = (q_1, 1, R)$, $\delta(q_1, \sqcup) = (q_2, \sqcup, L)$, $\delta(q_2, 1) = (q_3, \sqcup, R)$. For any other values $\delta$ is not defined.
Compute the output of $M$ executed on the configuration: 110111.

**Problem 18.** Write down explicitly a Turing machine $M$ over $\Sigma = \{0\}$ which computes the function $d : \mathbb{N} \to \mathbb{N}$ given by $d(n) = 2n$.
Use unary representation: A number $n$ is represented by the string $0^n$ consisting of $n$ copies of the symbol 0.

**Problem 19.** Construct a Turing machine $M = (Q, \Gamma, \sqcup, \{0, 1\}, \delta, q_0, F)$ such that $L(M) = \left\{ 1^k 0 1^{k+1} \,\middle|\, k \in \mathbb{N} \right\}$. Write down $Q$, $\Gamma$, $F$ and $\delta$ explicitly.

**Problem 20.** Write down explicitly an enumerator $G$ such that $\mathrm{Gen}(G) = \left\{ 0^{2n} \,\middle|\, n \in \mathbb{N} \right\}$.
Since in the lecture notes it has not been *formally* defined, how a Turing machine with two tapes works, you may describe the transition function as

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, L\} \times (\Gamma \cup \{\boxtimes\})$$

in the following way: If $G$ is in state $q$ and reads the symbol $c$ from the working tape, and

$$\delta(q, c) = (q', c', d, c'')$$

then $G$ goes to state $q'$, replaces $c$ by $c'$ on the working tape and moves the working tape head in direction $d$. Moreover, unless $c'' = \boxtimes$, the symbol $c''$ is written on the output tape and the output tape head is moved one position forward. If, however, $c'' = \boxtimes$, nothing is written on the output tape and the output tape head rests in place.
Hint: There exists a solution with only 4 states.