Problems Solved:

1	2	3	4	5

Name:

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Problem 1. Show by induction that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

for $n \geq 1$.

Problem 2. Let $L \subseteq \Sigma^*$ be a language over the alphabet $\Sigma = \{a, b, c, d\}$ such that a word w is in L if and only if it is either a or b or of the form $w = ducvd$ where u and v are words of L. For example, $dacad$, $ddacbdcad$, $dddbcbdcdbcbddcad$ are words in L. Show by induction that every word of L contains an even number of the letter d.
Note that a *language* is just a set of words and a *word* is simply a finite sequence of letters from the alphabet.

Problem 3. Show $\sqrt{\frac{2}{3}} \notin \mathbb{Q}$ by an indirect proof.
Hint: `http://en.wikipedia.org/wiki/Square_root_of_2#Proofs_of_irrationality`.
Hint: Note that the fact that $\sqrt{2}$ and $\sqrt{3}$ are irrational does not imply that $\frac{\sqrt{2}}{\sqrt{3}}$ is irrational as well. For example $\frac{\sqrt{2}}{\sqrt{8}}$ is rational.

Problem 4. Construct a deterministic finite state machine M over the alphabet $\{a, b, l, -\}$ such that it accepts the language $L(M) = \{bla\}$.

(a) Draw the graph.

(b) Provide the components of the defining quintuple $M = (Q, \Sigma, \delta, q_0, F)$ explicitly.

(c) What has to be changed in order for the machine to accept all finite strings of the form bla, $bla - bla$, $bla - bla - bla$ ...? (The empty word shall not be accepted.)

Problem 5. Construct a nondeterministic finite state machine for:

1. the language L_1 of all strings over $\{0, 1\}$ that contain 100 as a substring.
2. the language L_2 of all strings over $\{0, 1\}$ that contain the letters 1, 0, 0 in exactly that order. (Note that before, in between and after these three letters any number of other letters may occur).

Your two machines must not use more than 4 states. Moreover, they should only differ in their transition functions. Draw their transition graphs.